Asymptotic States of the Sum of Random Variables

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ABSTRACT
Most of the problems of probability theory are connected with the study of sums of random variables. The distribution of the sum of independent random variables is determined by the composition of the distributions of each additive. But this operation, defined in the class of distribution functions, is complicated, and in connection with this, efforts aimed at accurately calculating the distribution of the sum of random variables turn out to be useless. Therefore, problems of approximating the distribution of the sum of random variables with the help of asymptotic expressions are of great importance. In turn, the problems of approximation of distributions are connected with the limit theorems of probability theory.

Keywords: Probability theory, sequence of random variables, central limit theorem, Lindeberg condition, Lindeberg-Feller theorem, mathematical

Introduction

$(L) \Rightarrow (F) \Rightarrow (UAN)$-each of these conditions plays a specific role in whether or not MLT is appropriate.[1]

We now present two main results.

Theorem 1. [2] (Lindeberg). If the Lindeberg condition (L) is satisfied for a sequence of independent random variables, then an arbitrary $x \in R$ for (2.3.1) the central limit theorem is appropriate [3-11].

Theorem 2. [2] (Lindeberg-Feller). $\{\xi_n, n \geq 1\}$ condition (F) is satisfied for the sequence. Then condition (L) is necessary for MLT to be valid [12-21].

These theorems, along with their various corollaries, form an important part of probability theory.

The following examples illustrate the position of individual terms in the MLT.

Sequences of non-MLT random variables:

(a) $\xi_1, \xi_2, \ldots$ – sequence of independent random variables,

$P\{\xi_1 = 1\} = P\{\xi_1 = -1\} = 1/2,\ k \geq 2$ and for

$$P\{\xi_k = \pm 1\} = \frac{1}{2c},$$

$$P\{\xi_k = \pm k\} = \frac{1}{2k^2}\left(1 - \frac{1}{c}\right),\ P\{\xi_k = 0\} = 1 - \frac{1}{c} - \frac{1}{k^2}\left(1 - \frac{1}{c}\right)$$

, here $c > 1$

be defined by equations.[3] $\{\xi_n\}$ we answer the question of whether MLT is appropriate for the sequence. (L) We start by checking the Lindeberg condition.

$$a_k = M\xi_k = 0,\ \sigma_k^2 = M\xi_k^2 = 1,\ k \geq 1$$

because of

$$\frac{1}{s_n^2}\sum_{k=1}^{n}\int_{\{x:|x|\geq \varepsilon s_n\}} x^2 dF_k(x) = \frac{1}{n}\sum_{k=1}^{n} M\left\{\xi_k^2 I\left(|\xi_k| \geq \varepsilon\sqrt{n}\right)\right\}.$$

If n the $\varepsilon\sqrt{n}>1$ is chosen to satisfy the inequality, then

$$\frac{1}{n}\sum_{\left[\varepsilon\sqrt{n}\right]}^{n} k^2 P\left\{\left|X_k\right|=k\right\} \approx \frac{1}{n}(n-\varepsilon\sqrt{n})\left(1-\frac{1}{c}\right) \to 1-\frac{1}{c}>0$$

.

This means that the Lindeberg condition is not satisfied. However, it does not follow that the MLT is inappropriate since the Lindeberg condition is sufficient. $\left\{\xi_k\right\}$ that the MLT for the sequence is not valid and it can be proved as follows.[3] The following

$$P\left\{\frac{\left|\xi_k\right|}{s_n}\geq\varepsilon\right\}=P\left\{\left|\xi_k\right|\geq\varepsilon\sqrt{n}\right\}=\begin{cases}0, \text{ agar } k<\varepsilon\sqrt{n}, \\ \frac{1}{k^2}\left(1-\frac{1}{c}\right), \text{ agar } k\geq\varepsilon\sqrt{n}\end{cases}$$

from the relationship

$$\max_{1\leq k\leq n} P\left\{\frac{\left|\xi_k\right|}{s_n}\geq\varepsilon\right\}\leq\frac{1}{\varepsilon^2 n}\left(1-\frac{1}{c}\right)\to 0, \quad n\to\infty$$

it follows that the (UAN) condition is fulfilled [20-26].

Conclusion

So, in this case, according to Feller's theorem, the Lindeberg condition is necessary for the MLT to be valid, and therefore $\left\{\xi_k\right\}$ MLT is not appropriate for sequence.

Accordingly, we use conditions (L) and (F) to determine how the above results are related to MLT. In this regard, it is composed of unrelated random variables $\left\{\xi_n, n\geq 1\right\}$ we see the sequence, while, here $\xi_n \sim N(0,\sigma_n^2)$ $\sigma_1^2=1$ and $\sigma_k^2=2^{k-2}$, $k\geq 2$. $S_n=\xi_1+\ldots+\xi_n$ Total $MS_n=0$ to mathematical expectation and $s_n^2=DS_n=2^{n-1}$ has dispersion, n is optional because $\frac{\xi_k}{s_n}\sim N(0{,}1/2)$. So, $\frac{S_n}{s_n}\sim N(0{,}1)$ $\left\{\xi_n\right\}$ MLT is appropriate for sequencing [4].

Then

$$\lim_{n\to\infty}\max_{1\leq k\leq n}\frac{\sigma_k^2}{s_n}=\lim_{n\to\infty}\frac{2^{n-2}}{2^{n-1}}=\frac{1}{2}\neq 0$$

It is clear that and at the same time

$$\max_{1\leq k\leq n} P\left\{\frac{\left|\xi_k\right|}{s_n}\geq\varepsilon\right\}\geq P\left\{\frac{\left|\xi_n\right|}{s_n}\geq\varepsilon\right\}=1-\frac{1}{\sqrt{\pi}}\int_{-\varepsilon}^{\varepsilon} e^{-u^{2\frac{1}{2}}/2}du>0.$$

Thus, we conclude that Feller's condition (F) is not fulfilled. From this $\left\{\xi_n\right\}$ Since MLT is satisfied, Lindeberg's condition (L) also fails. It follows that the Lindeberg condition (L) is sufficient but not necessary for the MLT to be valid.

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