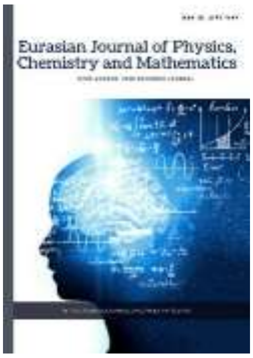

The Ranking Function for fuzzy parameters of Rayleigh Distribution

Rehab Noori Shalan
Department of mathematics, college of science, University of Baghdad, Baghdad, Iraq
Rehabnoori45@gmail.come

Sundos Bader Habeeb
Department of mathematics, college of science, University of Baghdad, Baghdad, Iraq.
badersundos@gmail.com

Rana A. Mohammed
Department of mathematics, college of science, University of Baghdad, Baghdad, Iraq.
rana.a@sc.uobaghdad.edu.iq

ABSTRACT

This research deals with estimating the parameters for the generalized Rayleigh distribution depending on the maximum likelihood estimator method.
Employing the real data to obtain the confidence intervals estimation for all parameters in generalized Rayleigh distribution. Fuzzy numbers will be construct from the trapezoidal membership function for this confidence interval of all parameters. Finally the ranking function algorithm using to convert the fuzzy numbers in to crisp (trapezoidal) fuzzy numbers and its applied to find the probability density function and survival function.

Keywords: Fuzzy number, Density function, Survival function, Maximum Likelihood estimation, Rank set estimator

1. Introduction

Rayleigh distribution is an interesting distribution in the field of statistic and operation research. It has multiple areas of applications such as health and biology, agriculture, and analyzing wind speed. In (2001) Surles and padgeth [18], suggested two parameters, Burr type X which are expressed as a generalization of Rayleigh distribution. It has been observed that this particular skewed distribution can be considered as a quite effectively and practically in life time data analysis.
Again Surles and Padgett in (2004)[19] considered the generalized Rayleigh distribution and discussed some different properties.
In (2005) Kundu and Raqab [13] discussed many modalities to estimate the parameters of Rayleigh distribution like maximum likelihood method, weighted least square, modified moment method, and moment method as well as they compared their interpretation using the Montecarlo simulation.
In (2009)[2], Al-Naqeeb and Hamed introduced a suitable method to facilitate estimation of the two- parameters of generalized Rayleigh distribution of different sample size (small, medium, and large).
Al-Qazaz in(2011)[3] reviewed method for estimating Rayleigh distribution with two-

parameters using the least squares regression method ,Ridge Regression method and modified Regression method.

In (2013)[15], Parvin, Ali, and Hossein investigated the estimation of the probability: $R = p(y < x)$, where x and y are random variables with two parameters of generalized Rayleigh distribution.

The main purpose of this research gives some estimation for the two parameters of Rayleigh distribution by employing maximum likelihood method by relying on Newton-Raphson method. Therefore, one concludes the interval estimation of two parameters. Then find the fuzzy number by utilizing $(\bar{x}+s^2)$ and $(\bar{x}-s^2)$ to make them trapezoidal member ship functions. After that applying Ranking function algorithm.

This research contained in section (2) the review of generalized Rayleigh distribution, section (3) gives some outlines of the fuzzy set theory, section (4) display the ranking function, and section (5) include some applications for this study.

2. The generalized Rayleigh distribution[[13]:

This distribution have two parameters which as follow

$$f(t,\lambda,\gamma) = \begin{cases} 2\gamma\lambda t e^{-\gamma t^2}\left(1-e^{\gamma t^2}\right)^{\lambda-1} & t \geq 0 \\ 0 & other\ wise \end{cases}$$

Where γ is shape parameter, **λ** is a scale parameter, the parameter space is $\Omega = \{(\lambda,\gamma), \lambda > 0, \gamma > 0\}$, this p.d.f. is called generalized Rayleigh distribution [1,7].

The cumulative distribution function for this distribution is:

$$F(t;\lambda,\gamma) = (1-e^{-\gamma t^2})^{\lambda}, for\ t \geq 1$$

Its survival function becomes as follows :

$$s(t;\lambda,\gamma) = 1-\left(1-e^{-\gamma t^2}\right)^{\lambda};\ for\ t \geq 1$$

Maximum likelihood estimator method (MLEM)[6]:

This procedure is mainly depend upon to estimate the parameter **λ** which is appoint a density probability function, f (t, λ) according the observations $t_1,\ldots,t_n$ these are independent sample for this distribution.

The maximum likelihood estimator $\hat{\lambda}$ corresponding to the parameter λ that maximize the likelihood function is given by:

$l(\lambda) = \prod_{i=1}^{n} f(t_i,\lambda)$

$$l(\lambda) = f(t_1,\lambda).f(t_2\lambda)\ldots f(t,\lambda)$$

The likelihood function for two –parameters generalized Rayleigh distribution is:

$$l(\lambda,\gamma,t_1,t_2,\ldots,t_n) = \prod_{i=1}^{n} f(t_i,\lambda,\gamma)\cdots\cdots\cdots(1)$$

$$l(\lambda,\gamma,t_1,t_{2,\ldots,}t_{n)=2^n\lambda^n\gamma^n\prod_{i=1}^{n}t_i}e^{-\gamma\sum_{i=1}^{n}t_i{}^2}.\prod_{i=1}^{n}(1-e^{-\gamma t_i{}^2})^{\lambda-1}\ldots\ldots\ldots(2)$$

Appling the logarithm to the likelihood function, one obtains:

$$\ln l = \mathrm{n}\ln 2 + n\ln\lambda + \mathrm{n}\ln\gamma + \sum_{i=1}^{n}\ln t_i - \gamma\sum_{i=1}^{n}t^2{}_i + (\lambda-1)\sum_{i=1}^{n}\ln(1-e^{-\gamma t^2{}_i})\cdots\cdots\cdots(3)$$

According to the unknown parameters λ and γ then the partial derivatives of the log-likelihood function will be formulated as:

$$\frac{\partial \ln l}{\partial\lambda} = \frac{n}{\lambda} + \sum_{i=1}^{n}\ln(1-e^{-\gamma t^2{}_i})\cdots\cdots\cdots(4)$$

$$\frac{\partial \ln l}{\partial\gamma} = \frac{n}{\gamma} - \sum t_i{}^2 + (\lambda-1)\sum_{i=1}^{n}\frac{t_i{}^2 e^{-\gamma t_i{}^2}}{\left(1-e^{-\gamma t_i{}^2}\right)}\cdots\cdots\cdots(5)$$

Replace λ with zero in (4), then: $\frac{\partial \ln l}{\partial\gamma} = 0$

$$\frac{n}{\hat{\lambda}} + \sum_{i=1}^{n}\ln(1-e^{-\hat{\gamma}t^2{}_i}) = 0\ldots\ldots\ldots\ldots..(6)$$

$$\hat{\lambda} = \frac{-n}{\sum_{i=1}^{n} \ln(1 - e^{-\hat{\gamma} t^2{}_i})} = 0 \ldots\ldots\ldots\ldots..(7)$$

Again substituting $\gamma = 0$ in (5), we get: $\frac{\partial \ln l}{\partial \gamma} = 0$

$$\frac{n}{\hat{\gamma}} - \sum t_i^2 + (\hat{\lambda} - 1) \sum_{i=1}^{n} \frac{t_i^2 e^{-\hat{\gamma} t_i^2}}{(1 - e^{-\hat{\gamma} t_i^2})} = 0 \cdots\cdots\cdots (8)$$

$$\hat{\gamma} = \frac{n}{\sum t_i^2 + (\hat{\lambda} - 1) \sum_{i=1}^{n} \frac{t_i^2 e^{-\hat{\gamma} t_i^2}}{(1 - e^{-\hat{\gamma} t_i^2})} \cdots\cdots\cdots (9)}$$

Observing that the two equations (7) and (9) are difficult and complicated to solve, then it is impossible to find MLE for λ and γ directly,

We use the numerical analysis (numerical procedure) to obtain and estimate λ and γ to maximize the like hood function, one thus numerical procedure is Newton-Raphson method, which can be written as follows:

$$g_1(\lambda) = \frac{n}{\lambda} + \sum \ln(1 - e^{-\gamma t_i^2}) \cdots\cdots\cdots (10)$$

$$g_2(\lambda) = \frac{n}{\gamma} - \sum t_i^2 + (\lambda - 1) \sum_{i=1}^{n} \frac{t_i^2 e^{-\gamma t_i^2}}{(1 - e^{-\gamma t_i^2})} \cdots\cdots\cdots (11)$$

According to the unknown parameter λ, , the partial derivative of $g_1(\lambda)$ is given by:

$$\frac{\partial g_1(\lambda)}{\partial \gamma} = \frac{-n}{\lambda^2} \cdots\cdots\cdots (12)$$

$$\frac{\partial g_1(\lambda)}{\partial \gamma} = \sum \frac{t_i^2 e^{-\gamma t_i^2}}{(1 - e^{-\gamma t_i^2})} \cdots\cdots\cdots (13)$$

The partial derivatives of $g_2(\gamma)$ with respect to unknown parameters λ and γ are:

$$\frac{\partial g_2(\gamma)}{\partial \gamma} = \sum \frac{t_i^2 e^{-\gamma t_i^2}}{(1 - e^{-\gamma t_i^2})} \ldots (14)$$

$$\frac{\partial g_2(\gamma)}{\partial \gamma} = \frac{-n}{\gamma^2} + (\lambda - 1) \sum_{i=1}^{n} \frac{(1 - e^{-\gamma t_i^2})(-t_i^4 e^{-\gamma t_i^2}) - (t_i^2 e^{-\gamma t_i^2})(t_i^2 e^{-\gamma t_i^2})}{(1 - e^{-\gamma t_i^2})^2}$$

$$\frac{\partial g_2(\gamma)}{\partial \gamma} = \frac{-n}{\gamma^2} - (\lambda - 1) \sum \frac{t_i^4 e^{-\gamma t_i^2}}{(1 - e^{-\gamma t_i^2})^2} \cdots\cdots\cdots (15)$$

$$J_k = \begin{bmatrix} \frac{\partial g_1(\lambda)}{\partial \lambda} & \frac{\partial g_1(\lambda)}{\partial \gamma} \\ \frac{\partial g_2(\gamma)}{\partial \lambda} & \frac{\partial g_2(\gamma)}{\partial \gamma} \end{bmatrix} \cdots\cdots\cdots (16)$$

The Jacobin matrix obtained in the above equation is a non- singular symmetric matrix, hence its inverse obtained from:

$$\begin{bmatrix} \lambda_{k+1} \\ \gamma_{k+1} \end{bmatrix} = \begin{bmatrix} \lambda_k \\ \gamma_k \end{bmatrix} - J_k^{-1} \begin{bmatrix} g_1(\lambda) \\ g_2(\gamma) \end{bmatrix}; \quad i = 1,2, \ldots\ldots\ldots (17)$$

Some procedure of iterations to express the error term, that structured according to the new values λ, γ and compute the absolute value for the difference between the, it is symbol is ∈,a very small value.

Then, error term is represented by:

$$\begin{bmatrix} \in_{k+1}(\lambda) \\ \in_{k+1}(\gamma) \end{bmatrix} = \left\| \begin{bmatrix} \lambda_{k+1} \\ \gamma_{k+1} \end{bmatrix} - \begin{bmatrix} \lambda_k \\ \gamma_k \end{bmatrix} \right\|$$

Where λ_k and γ_k are the assumed initial values.

3. Fuzzy set theory[4] :

Realistic and vital life require to create the Fuzzy sets which are a generalization of the crisp sets in ordinary set theory.

Fuzzy sets were introduced by Zadeh in 1965 as a mathematical method to represented the fogginess in many life and physical phenomena, like heat, velocity, tall measure, and others[5]. This is the generalization of crisp sets in terms of membership function, the notion of Fuzzy set $\breve{B}$ on the universe of discourse U is the set of order pair $\{(y, \mu_{\tilde{B}}(y), y \in U\}$ with member ship function $\mu_{\tilde{B}}(y)$ taking the value on the interval[0,1][14]

Definition [17]: A Fuzzy set $\check{B}$ is set of members associated with a Y class of events with a grade of membership function denoted by $\mu_{\tilde{B}}(y)$ which associates for each element $y \in Y$ a value in the interval [0, 1].

"Traditionally, the grade of member ship 1 is assigned to those object (event Y), that fully and completely belong to B, while 0 is assigned to objects (event y)that does not belong to B at all, Moreover, an object y belong to B closer to 1 be its grade of member ship $\mu_{\tilde{B}}(y)$"[6]

Definition [8]:" The support of a fuzzy set $\tilde{B}$ (denoted by supp$(\widetilde{B})$ is the crisp set of all $y \in Y$ such that$\mu_{\tilde{B}}(y) > 0$".

Definition [8]: The greatest member ship value is called the height of $\tilde{B}$, i.e., $hgt(\tilde{B}) = \sup y \in Y \ \mu_{\tilde{B}}(y)$

Definition [8]: $\tilde{B}$ is said to be normal if and only if there exists $y \in Y$ such that $\mu\tilde{B}(Y) = 1$, other wise $\tilde{B}$is subnormal.

The main and interesting concept related to the object what are called λ- level or λ- cut sets these are relate between fuzzy and ordinary sets, as well as they may be used to prove some results for fuzzy sets which are has been proved in ordinary set theory **[6]**.

Definition [18]: Let Y be any universal set The λ-level (or λ- cut) set of a fuzzy set $\tilde{B}$ denoted by B_λ, is the crisp set of all elements y in Y such that $\mu_{\tilde{B}}(y) \geq \lambda$; i.e,

$$B_{\lambda =}\{y \in Y | \mu_{\tilde{B}}(y) \geq \lambda, \lambda \in [0,1]\}.$$

Definition [24]: A fuzzy number of $\tilde{B} = (a, b, c, d)$ is called a trapezoidal fuzzy number if its member ship function is given by[16]:

$$\mu_{\tilde{B}}(y) = \begin{cases} \dfrac{y-a}{b-a} & a \leq y \leq b \\ 1 & b \leq y \leq c \\ \dfrac{y-d}{c-d} & c \leq y \leq d \end{cases}$$

4. Ranking function[9]:

The method for ranking was first proposed by Jain (1976) [12.20]. Yager (1981) [20] propose of ordering fuzzy amounts in [0, 1]. Ghen and Ghen (2007) [5, 20] presented a method for ranking generalized trapezoidal fuzzy number generalized fuzzy number.

Yager introduced the ranking function as a relatively simple computationally and comprehensible ranking of fuzzy number. He suggested a procedure to order the fuzzy sets, where the ranking index $R(\tilde{B})$ is computed for the fuzzy numbers.

$\tilde{B} = (a, b, c)_{L_R}$ from the extreme value of it is λ-cut

$B_\lambda = [b\lambda^4 + aL(x), c\lambda^2 + dR(x)]$ according to the following formula:

$$R(\tilde{B}) = \frac{1}{2}\int_0^1 \left(\inf \widetilde{B_\lambda} + \sup \tilde{B}_\lambda\right) d\,\lambda$$

We put $\frac{1}{2}$ outside the integral $R(\tilde{B})$, because $\frac{1}{2}$ is the weight that means giving weight for $\inf \tilde{B}_\lambda = \frac{1}{2}$ and weight for $\sup \tilde{B}_\lambda = \frac{1}{2}$

$$R(\tilde{B}) = \frac{1}{2}\int_0^1 (\lambda^4 b + aL(y) + \lambda^2 c + dR(y))d\lambda$$

Suppose that $\tilde{B} = (a, b, c)_{L_R}$ is an L-R flat fuzzy number

$L(y) = Max\ (0,1 - y^4), R(y) = Max(0,1 - y^2), y < 1$ to find $R(\widetilde{B})$ by applying the following formula

$$\frac{y-a}{b-a} = \lambda^4 \quad , \qquad y - a = \lambda^4(b-a)$$

$$y=\lambda^4(b-a)+a=\inf\tilde{B}_\lambda, \qquad \frac{y-d}{c-d}=\lambda^2$$
$$y-d=\lambda^2(c-d), \qquad y=\lambda^2(c-d)+d=\sup\tilde{B}_\lambda$$
$$R(\tilde{B})=\frac{1}{2}(\int_0^1 \lambda^4 b+a(1-\lambda^4))d\lambda+\int_0^1(\lambda^2 c+d(1-\lambda^2)d\lambda)$$
$$R(\tilde{B})=\frac{1}{2}\int_0^1(\lambda^4 b+a-a\lambda^4+\lambda^2 c-d\lambda^2+d)\,d\lambda$$
$$g(\tilde{B})=\frac{1}{2}\left[\frac{\lambda^5 b}{5}+a\lambda-\frac{\lambda^5}{5}a+\frac{\lambda^3}{3}c-\frac{\lambda^3}{3}d+d\lambda\right]_0^1$$
$$g(\tilde{B})=\frac{1}{2}\left[\frac{b}{5}+\frac{4a}{5}+\frac{c}{3}+\frac{2d}{3}\right]$$
$$g(\tilde{B})=\frac{1}{30}[3b+12a+5c+10d]\text{.................} (18)$$

5. Applications [10,11]:

The real data for breast cancer disease for woman has been Chosen because it is widespread in Iraq. Depending on the data for this disease from the Hospital of Radiation and Nuclear Medicine for period from 1/7/2021 until 31/12/2021, the number of patient in this time is (104), twenty-four patients are dead and eighty remained alive, this means that the complete data for the dead patients became :

T= [5, 7, 9 ,11,15,19, 22,26,30, 35 ,39,42,44,50,35,58,60,63, 65, 66,71 ,73, 78, 79]

Maximum likelihood estimator method

The estimate values of two parameters of Rayleigh distribution when use the (mathlap) program by employing the equation (17) are given by:

$$\hat{\lambda}=0.06134 \qquad \hat{\gamma}=0.003517$$

The estimated confidence interval obtained by applying the following formula as follows:

$$\left[\hat{\lambda}-t_{(n-1.1-\alpha)}\sqrt{var(\hat{\lambda})},\hat{\lambda}+t_{(n-1.1-\alpha)}\sqrt{var(\hat{\lambda})}\right]=[\hat{\lambda}_1,\hat{\lambda}_1], \quad \left[\hat{\gamma}-t_{(n-1.1-\alpha)}\sqrt{var(\widehat{\gamma})},\hat{\gamma}+t\sqrt{var(\widehat{\gamma})}\right]$$
$$=[\hat{\gamma}_1,\hat{\gamma}_2]$$

Then the Interval estimation for real data of a parameter $\hat{\lambda}$ is given by:

$$var(\hat{\lambda})=0.000045 \qquad ,\sqrt{var(\hat{\lambda})}=0.00671$$
$$t(n-1,1-\alpha)=t(23,0.95)=1.714 \qquad , \qquad [\hat{\lambda}_1,\hat{\lambda}_1]=[0.04984,0.07284]$$

As well as, the Interval estimation for real data of a parameter $\hat{\gamma}$ becomes:

$$var(\hat{\gamma})=0.00000057988 \quad , \quad \sqrt{var(\hat{\gamma})}=0.0007615, \quad t(n-1,1-\alpha)=t(23,0.95)=1.714$$
$$[\hat{\gamma}_1,\hat{\gamma}_2]=[0.00221178,0.004822]$$

After that fuzzing the confidence interval estimation by utilizing the trapezoidal membership function as follows:

$$\hat{\lambda}=\left[\hat{\lambda}-t_{(n-1.1-\alpha)}\sqrt{var(\hat{\lambda})},\bar{x}-\widehat{s^2},\bar{x}+\widehat{s^2},\hat{\lambda}+t_{(n-1.1-\alpha)}\sqrt{var(\hat{\lambda})}\right]$$

$$\hat{\gamma}=\left[\hat{\gamma}-t_{(n-1.1-\alpha)}\sqrt{var(\hat{\gamma})},\bar{x}-\widehat{s^2},\bar{x}+\widehat{s^2},\hat{\gamma}+t_{(n-1.1-\alpha)}\sqrt{var(\hat{\gamma})}\right]$$
$$\hat{X}=\frac{lower\ interval\ limit+upper\ interval\ limit}{2}$$
$$\widehat{s^2},=\frac{(lower\ interval\ limit+\hat{X})^2+(upper\ interval\ limit+\hat{X})^2}{2}$$

Then the fuzzy numbers of two parameters of Rayleigh distribution, when applying trapezoidal membership function becomes:

$\tilde{\lambda} = [0.04984, 0.0612072, 0.0614722, 0.07284] = [a, c, d, b]$

$\tilde{\gamma} = [0.00221178, 0.0035151866, 0.00351859, 0.004822] = [a, c, d, b]$

Employing the ranking function from the equation (18) to find the crisp numbers (optimization number)

$R(\tilde{B}) for\ \tilde{\lambda} = 0.0579, \quad R(\tilde{B}) for\ \tilde{\gamma} = 0.0031255$

Finally applying the failure time of real data for the ranking function of $\hat{\lambda}$ and ranking function of $\hat{\gamma}$ to find the probability density function f (t), as well as the survival function s (t) and tableting from the following table:

Table (1) represents the f (t) and s (t)

T	f(t)	S(t)
5	0.0191681981	0..1391637376
7	0.01367248396	0.1068647302
9	0.0103674969	0.08305933672
11	0.008101264819	0.06471509445
15	0.005114701749	0.03878274954
19	0.003216018722	0.02238132079
22	0.002217649833	0.01430525632
26	0.00128447734	0.00742668411
30	0.0006909218681	0.003577873088
35	0.000281117776	0.001271645824
39	0.0001226444261	0.0005010276171
42	0.0000615317377	0.000233923306
44	0.00003759617362	0.0001365396895
50	0.000007316357961	0.00002340414316
53	0.00000295165551	0.000008908586967
58	0.0000005699226798	0.00000157192888
60	0.0000002819616049	0.000000751772725
63	0.00000009343286913	0.000000237251404
65	0.00000004330922318	0.000000106590169
66	0.00000002920068034	0.000000070778155
71	0.000000003692296517	0.00000000831933
73	0.000000001543240013	0.0000000033819
78	0.0000000001557319446	0.00000000031939
79	0.00000000009656069256	0.00000000019553

6. Conclusions

1. From table (1) showing that the f (t) are increasing with failure time (t) after reaching the mode which is (t=44) then starting to be decreasing with the failure time.
2. showing the s (t) are decreasing with failure time.

References

1. AL-Khedairi A., Ammar M.S., and TAdj L. (2008), "Estimation of the generalized Rayleigh distribution parameters", international journal of Reliability and applications,Vol.7,No.1, PP.1-12
2. AL- Naqeeb A.A. and Hamed A.M. (2009), " Estemation of the two parametars for generalized Raylieh Distribution function using simulation technique", IBN AL-

Haitham J. for pure and applied Sci., Vol. 22(4).

3. Al-Qazaz Q.N. (2011),” Comparing the approximate Estimators of Reliability for Rayligh distribution with two parameters”, Al-Ustath journal-Education college, Ibn Rushd, PP.1-12.
4. Arefi M.,Viertl R.and Taheri S.M.;2009:Fuzzy Density Estimation ;Technische Universitat Wien;Vol.1;Pp.1-17
5. Bezdek J.C. (1993),” Fuzzy model – what are they and why?”, IEEE transections on fuzzy systems, Vol. 1, No. 1, PP. 1-6.
6. Douglas C.M. and George C.R. (2003),"Applied Statistics and probability for Engineering", Third Edition ,John Wiley and Sons ,Inc.
7. Essam A. A.(2011), “ Maximum likelihood estimation of the mixed generalized Rayleigh distribution from type I censored samples”, Applied mathematical science, Vol. 5, No.56, PP. 2753-2764.
8. George J. K. and Tina A. F.(1988), “ Fuzzy sets uncertainty and information”, Prentice Hall, New jersey.
9. Iden H.H. and Esraa D.T.(2021), " Solving fuzzy attribute quality control charts with proposed ranking function", Ibn Al-Haitham jpornal for pure and applied science,
10. Iden H.H. and Hadeer A.K. (2019), "Estimation of survival functions for Rayleigh distribution by ranking function", Baghdad science journal, Vol. 16 No. 3, PP. 775-780.
11. Iden H.H. and Hadeer A.K. (2019), "Fuzzy Survival and Hazard functions estimation for Rayleigh distribution", Iraqi journal of science, PP. 624-632.
12. Jain R. (1976), "Decision making in the presence of fuzzy variables", IEEE Transaction on systems Man and Cybernetics, Vol. 6, PP. 698-703.
13. Kundu D. and Raqab M.Z. (2005), “ Generalized Rayleigh distribution :different methods of estimation) statistics and data analysis, 49, PP.187-200
14. Nilamber S., das K., and Panda D.C.(2012), “ probabilities interpretation of complex fuzzy set”, International journal of computer science, engineering and ,information technology, Vol. 2, No.2, PP. 31-44.
15. Parvin F. Ali A., and Hussein J.K. (2013), “Estimating R=P(Y<X) in the generalized Rayleigh distribution with different scle parameters”, applied mathematical science, vol. 7, No.2, PP. 87-92
16. Kumar.A.,Singh.P. ,Kaur.A.and Kaur.P.;2010;Ranking of Generalized Trapezodial Fuzzy NUMbers Based onRank,Mode,Divergence and Spread; Turkish Journal of fuzzy System;Vol.No.2;1;Pp141-52.
17. Saeed N.H.(2012), “ Fuzzy- parametric linear programming problem in general company for electrical industrials, master thesis, Ibn –Al Haitham collage of education, Baghdad university.
18. Surles J.G and Padgett W.J. (2001)," Inference for reliability and stress-strength for a scaled Burr Type X distribution", Lifetime data analysis, Vol.7, PP.187-200.
19. Surles J.G., and Padgett W.J. (2004), “ Some properties of a scale burr type X distribution”, to appear in the journal of statistical planning and inference.
20. Yager R.R. (1981), " A procedure for ordering fuzzy subsets of the unite interval" Information science, Vol. 24, No. 2, PP. 143-161.