

*Issue 1*. for the numbers  $\forall$  $x, y, z \in (0;1)$ 

ABC with a side equal to 1, we place points M, K, N on its sides AB, BC, CA in such a way that AM=x, BK=z, CN=y (Fig. 1)



**Figure 2.**1. A form suitable for the condition of the matter.

If we assume that the surfaces of the resulting triangles AMN, CNK, BMK are  $S_1, S_2, S_3$  corresponding, then

 $(1 - y);$ 4 3  $S_1 = \frac{\sqrt{3}}{4}x(1-y);$   $S_2 = \frac{\sqrt{3}}{4}y(1-z);$ 4 3  $S_2 = \frac{\sqrt{3}}{4}y(1-z);$  $(1 - x)$ 4 3  $S_3 = \frac{36}{4}z(1-x)$  will be the result.

 $S_1 + S_2 + S_3 < S_{ABC}$  and  $S_{ABC} = \frac{S_3}{4}$  $S_{ABC} = \frac{\sqrt{3}}{4}$ 

it is known. Based on this 3 3 3

$$
\frac{\sqrt{3}}{4}x(1-y) + \frac{\sqrt{3}}{4}y(1-z) + \frac{\sqrt{3}}{4}z(1-x) < \frac{\sqrt{3}}{4}
$$

$$
\frac{\sqrt{3}}{4}(x(1-y)+y(1-z)+z(1-x)) < \frac{\sqrt{3}}{4}
$$

 $x(1 - y) + y(1 - z) + z(1 - x) < 1$ originates. The inequality is proved *Issue 2*. It is known that x+y=6 for positive numbers x and y. *<sup>x</sup> y*  $\frac{1}{-} + \frac{1}{-}$  find the smallest value of the expression.

*Solution*: We are given the equation x+y=6. If we multiply both parts by 2, we get 2x+2y=12. We can make a rectangle with a perimeter of 12. We change the form of the required sum of *<sup>x</sup> y*  $\frac{1}{-} + \frac{1}{-}$  as follows:

we change the sum as follows: .  $1 + 1 = x + y = 6$ *xy xy x y x y* = +  $+ - =$ 

It is known that the square with side 3 has the largest area among rectangles with a perimeter of 12. This is because the face of the square is equal to 9 3 2 9  $1 \t1 \t x+y \t6 \t6$  $=\frac{0}{0}=\frac{0}{0}$ +  $+ - =$ *xy xy x y*  $\frac{y}{x} + \frac{z}{y} = \frac{z}{xy} = \frac{z}{xy} = \frac{z}{9} = \frac{z}{3}$  will be the result. So, the smallest value of the given expression *<sup>x</sup> y*  $\frac{1}{-}{+}\frac{1}{-}$  is equal to .

*Issue 3*. If the following conditions are true for positive numbers x, y, z, a, b, c, find the value of the sum xy+yz+zx.

$$
\begin{cases}\nx^2 + xy + y^2 = a^2 \\
y^2 + yz + z^2 = b^2 \\
z^2 + zx + x^2 = c^2\n\end{cases}
$$

## **Volume 11| October 2022 ISSN: 2795-7667**

*Solution*: In addition to surface formulas, we use the theorem of cosines to solve this problem. We place three cross-sections with lengths x, y, z in such a way that they have a common point O and make an angle of 120° with each other. By connecting the other ends of these sections, we form a triangle ABC. We take the lengths of the sides of this triangle as a, b, c, respectively (Fig. 2.2). Based on the theorem of cosines *x <sup>2</sup>+xy+y2=a2, y2+yz+z2=b2, z2+zx+x2=c<sup>2</sup>*  equalities will be appropriate.



*Figure 2.2*. A form suitable for the condition of the matter

Now if we call the surfaces COB, AOC and AOB formed  $_1$ ,  $\omega_2$  $S_1$ ,  $S_2$ ,  $S_3$ , then  $S_1 = \frac{V}{4}xy$ 3  $s_2 = \frac{\sqrt{3}}{4}yz$ <br> $s_2 = \frac{\sqrt{3}}{4}yz$ 3  $2=\frac{\sqrt{3}}{4}yz;$ 

 $S_2 = \frac{1}{2}zx$ 4 3  $S_3 = \frac{C}{4}zx$  equalities will be appropriate.

 $S_1 + S_2 + S_3 = S_{ABC}$  from the fact that originates.

 $\frac{3}{2}(xy+yz+zx)$  $\frac{d^2y}{dx^2}(xy + yz + zx) = S_{ABC}$  On the other hand, we find  $S_{ABC}$  - the face of a triangle using Heron's formula:  $S = \sqrt{p(p-a)(p-b)(p-c)}$ formula

$$
S = \sqrt{\frac{a+b+c}{2} \cdot \frac{a+b-c}{2} \cdot \frac{a+c-b}{2} \cdot \frac{b+c-a}{2}}
$$

$$
\frac{\sqrt{3}}{4}(xy + yz + zx) =
$$
\n
$$
= \frac{1}{4}\sqrt{(a+b+c)(a+b-c)(a+c-b)(b+c-a)}
$$
 and from that  
\n
$$
xy + yz + zx = \sqrt{\frac{(a+b+c)(a+b-c)(a+c-b)(b+c-a)}{3}}
$$
 equalities will be

appropriate.

*Issue 4*. If  $y > 0$ ,  $x + y^2 = 7.25$ ,  $y^2 - z = 2$  and  $y^2 = \sqrt{x-1} \cdot \sqrt{2-z}$  if equalities are appropriate  $y \cdot (\sqrt{x-1} + \sqrt{2-z})$  find the value of the expression. **The solution**. First,  $x \neq 1$ ,  $z \neq 2$ . In fact, if x=1 or z=2, then y=0. Any pair of 1's and 0's  $x + y^2 = 7.25$  does not satisfy the condition. It can be seen that the pair of numbers 0 and 2 does not satisfy the relation  $y^2 - z = 2$ . Secondly, for the numbers x>1 and z<2, we describe conditions  $x + y^2 = 7.25$  and  $y^2 - z = 2$ as follows:  $(\sqrt{x-1})^2 + y^2 = 6,25$  $(x-1)^2 + y^2 = 6,25$  and  $y^2 + (\sqrt{2-z})^2 = 4$ . now we can draw a corresponding drawing (Fig. 2.3).



*Figure 2.3*. A form suitable for the condition of the matter. Thirdly, according to the formula for finding the face of this triangle, we write the following equation:

$$
S = \frac{1}{2} y \cdot (\sqrt{x-1} + \sqrt{2-z}) = \frac{1}{2} \cdot 2 \cdot 2,5
$$
  

$$
y \cdot (\sqrt{x-1} + \sqrt{2-z}) = 5
$$

So, the value of the search expression is equal to 5. *Issue 5*. x, y, z for positive numbers

$$
\begin{cases}\nx^2 + xy + \frac{y^2}{3} = 25 \\
\frac{y^2}{3} + z^2 = 9 \\
z^2 + zx + x^2 = 16\n\end{cases}
$$

calculate the value of the sum xy+2yz+3xz without solving the system of equations. **Solution**: We will draw a diagram corresponding to the condition of the problem (Fig. 2.4).



*Figure 2.4*. A form suitable for the condition of the matter. As can be seen from the diagram,

$$
S_{\Delta ABC} = \frac{1}{2} x \cdot \frac{y}{\sqrt{3}} \sin 150^\circ + \frac{1}{2} \cdot \frac{y}{\sqrt{3}} z + \frac{1}{2} x z \sin 120^\circ = \frac{1}{2} x \cdot \frac{y}{\sqrt{3}} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{y}{\sqrt{3}} z + + \frac{1}{2} x z \cdot \frac{\sqrt{3}}{2} = \frac{1}{4\sqrt{3}} (xy + 2yz + 3xz)
$$

On the other hand, the face of a right-angled triangle ABC is equal to 6. Accordingly  $(xy + 2yz + 3xz) = 6$  $4\sqrt{3}$ 1  $xy + 2yz + 3xz$ ) from that  $(xy + 2yz + 3xz) = 24\sqrt{3}$ we determine that. Answer:  $(xy + 2yz + 3xz) = 24\sqrt{3}$ 

*Issue* 6. If  $x > 0$ ,  $y > 0$ ,  $z > 0$  and  $xyz(x + y + z) = 1$  if  $(x + y)(x + z) \ge 2$  prove the inequality.

**Solution**: since  $x>0$ ,  $y>0$ ,  $z>0$ , there is a triangle ABC whose sides are AB=c=x+y,  $BC=a=v+z$ ,  $AC=b=x+z$  (Figure 2.5).



*Figure 2.5*. A form suitable for the condition of the matter.

The circle inscribed in this triangle corresponds to the sides AB, BC, AC try at points K, M, N. In this case,  $x+y+z=p$ , where p is the semiperimeter. In addition, AK=AN=p-a=x, BK=BM=p-b=y, CM=CN=p-c=z relations are appropriate. But as the case may be,  $xyz(x+y+z)=p(p-a)(p-b)(p-c)=S^2=1$  or S=1, where S is a face of triangle ABC. On the other hand,  $2S = AB \cdot AC \sin BAC \le AB \cdot AC = (x + y)(x + z)$ 

will be. *(x+y)(x+z) 2S=2* Then the Inequality is proved.

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