

On Some Types Nano – M – Continuous Functions

Nawras Hasan Mohammed¹

^{1,2}Department of Mathematics, College of Education for Pure Sciences, Tikrit University, Tikrit, Iraq Salahaddin Tikrit
E-mail: nawras.h.mohammed@st.tu.edu.iq

Ali A. Shihab²

^{1,2}Department of Mathematics, College of Education for Pure Sciences, Tikrit University, Tikrit, Iraq Salahaddin Tikrit
E-mail: draliabd@tu.edu.iq, ali.abd82@yahoo.com

ABSTRACT

The aim of this work is to definition new type of Nano- M -continuous function. It is named Nano-M-(Nano semi, Nano pre_ , Nano α _ , Nano β _ Nano δ – Nano θ _ , Nano θ _ , Nano Re_ , Nano θ _S_) continuous function and to reach a relationship with types of Nano- M - cont function are studied with some examples, properties and necessary theorems are studied on it.

Keywords:

Nano-Continuous function , Nano – M – Continuous function

1. Introduction

In 2013 Thivagar M. Lellis [10] idea of Nano-topological space (N_TS) with respect to a subset X of universe U which is defined as an upper and lower approximation of x . The element of N_TS is called a Nano-open sets(N_os).

In 1963 levine [2] establish the notion of semi-cont. function. In 1983 M.E. Abd EL-Monsef [1] establish the notion of β -cont. function, α -cont function [12], pre-cont. function [9], δ - cont. function, θ – semi cont. function, Regular-cont. function [3], θ – cont. function. In 2022 [4] Mohmmmed N.H and Shihab A. A and we will intrudes new type of Nano-cont. function and introduced definition of Nano-M - open set.

2. Preliminaries

A subset A of a space (X, T) is called semi-open(se_o) [10] (resp. α -open, β -open, δ -open, θ -open, Re_o) [7], pre open(pr_o) [10], δ –

open(δ_o) [6], θ -open(θ_o) [8], Regular-open(Re_o) [10], θ -semi-open(θ_S_o) [6] set. The complement of se_o (resp., α_o , β_o , pr_o , δ_o , θ_o , Re_o , θ_S_o) set is said to be semi-closed(se_c) (resp., α -c, β -c, pre-c, δ -c, θ -c, Regular-c, θ_S -c) set. Intersection of all se_c (resp. α - c, β - c, pre - c, δ - c, θ - c, Regular- c, θ_S -c) sets continuing A is called the semi- closure (resp. α - closure, β - closure, pre -closure, δ - closed, θ - closure, Regular- closure, θ - semi - closure) and is denoted by $Scl(A)$ [resp. $\alpha cl(A)$, $\beta cl(A)$, $pcl(A)$, $\theta cl(A)$, $Rcl(A)$, $\delta cl_\theta(A)$].

The union of all se_o (resp. α_o , β_o , pr_o , δ_o , θ_o , Re_o , θ_S_o) sets contained in A is said semi- interior (resp. α - interior, β - interior, pre interior, δ - interior, θ - interior, Regular- interior, θ - semi - interior) and denoted $\alpha int(A)$ [resp. $\alpha int(A)$, $\beta int(A)$, $pint(A)$, $\theta int(A)$, $Rint(A)$, $\delta int_\theta(A)$].

The family of all semi-cont (resp., α -cont, β -cont, pre-cont, δ -cont, θ -cont, Regular-cont, θ -semi-continuous, θ -continuous) is denoted by $\delta\text{cont}(x)$, [resp., α -cont(x), β -cont(x), θ -cont(x), R-cont(x), δ -cont(x)].

Definitions 2.1.1:[10] Let $(U, T_R(x))$ be a N_{TS} and $A \subseteq U$. Then A be called.

- 1- N_α -o. if $A \subseteq N\text{int}(N\text{cl}(N\text{int}(A)))$.
- 2- N -pre-o. if $A \subseteq N\text{int}(N\text{cl}(A))$.
- 3- N_δ -o. if $A \subseteq N\text{cl}(N\text{int}(A))$.
- 4- $N_{\theta S}$ -o. if $A \subseteq \overline{A_\theta^0}$. [6]
- 5- N_R -o. if $A = N\text{int}(N\text{cl}(A))$.
- 6- N_θ -o. if $A = \overline{NA_\theta^0}$. [8]
- 7- N_δ -o. if $A \subseteq \overline{A_\delta^0}$. [6]
- 8- N_β -o. if $A \subseteq N\text{cl}(N\text{int}(N\text{cl}(A)))$. [7]

3. Some Types of M-N-continuity

Definition 3.1:[4] let $(U, T_R(x))$ be N_{TS} . The subset A of U is called MN-open set (MN - o.s.) in N_{TS} if $A \subseteq \overline{NA_\theta^0} \cup \overline{NA_\delta^0}$.

Remark 3.2: The complement of MN - o.s. is said to be MN - closed set(MN - c.s.)

Definitions 3.3: Let $(U, T_R(x))$ and $(Q, T_R(H))$ be N_T spaces. A function $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ is called:

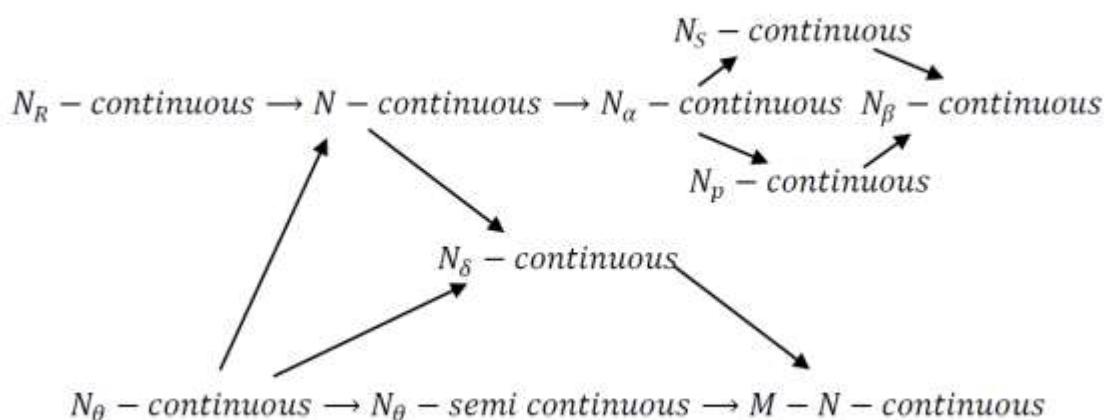
- 1- Nano-continuous(N - Con) if $f^{-1}(A)$ is N -open set in U , $\forall N$ -open set A in Q . [11]

- 2- $Nano\alpha$ - continuous (N_α - Con) if $f^{-1}(A)$ is N_α -open set in U , $\forall N$ -open set A in Q . [12]
- 3- N pre - continuous(N - pre - Con) if $f^{-1}(A)$ is N -pre-o.s. in U , $\forall A$ is N - pre - o.s. in Q . [9]
- 4- N_β -continuous(N_β - Con) if $f^{-1}(A)$ is N_β - o.s. in U , $\forall A$ is N_β - o.s. in Q . [5]
- 5- N_S -continuous(N_δ - Con) if $f^{-1}(A)$ is N_S - o.s. in U , $\forall A N_S$ - o.s. is in Q . [13]
- 6- N_R -continuous (N_R - Con) if $f^{-1}(A)$ is N_R - o.s. in U , $\forall A$ is N_R - o.s. in Q . [3]
- 7- N_θ -continuous(N_θ - Con) if $f^{-1}(A)$ is N_θ - o.s. in U , $\forall A$ is N_θ - o.s. in Q .
- 8- N_θ -semi-continuous($N_{\theta S}$ - Con) if $f^{-1}(A)$ is $N_{\theta S}$ - o.s. in U , $\forall A$ is $N_{\theta S}$ - o.s. in Q .
- 9- N_δ -continuous(N_δ - Con) if $f^{-1}(A)$ is N_δ - o.s. in U , $\forall A$ is N -open set in Q .

Definition 3.4: let $(U, T_R(x))$ and $(Q, T_R(H))$ be M-Nanotopological space (MN - TS). Then a function $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ is M_N -continuous (M_N - Con) on U if $f^{-1}(A)$ is MN - o.s. in U , $\forall A$ is MN - o.s. in Q .

Remark 3.5: The following diagram explains the relations between continuous functions

Diagram (1)



The relationship between continuous functions

Example 3.6: Let $U = \{a, b, c, d\}$ with $T_R(x) = \{U, \emptyset, \{a, b, c\}\}$, then MN - o.s. (U) = $\{U, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ and let $Q = \{1, 2, 3, 4\}$ with $T_R(H) = \{Q, \emptyset, \{1, 2, 3\}\}$, then

MN - o.s. (Q) = $\{Q, \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$. Define $f: U \rightarrow Q$ as $f(a) = 1, f(b) = 2$ and $f(c) = 3$. Therefore f is M_N -cont. mapping on U , so the above diagram (1) achieves it.

Theorem 3.7: A function $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ is M_N - Con $\Leftrightarrow f^{-1}(A)$ MN - c. s. in Q is MN - c. s. in U .

Proof: h is M_N - Con, H is MN - c. s. in Q . That is, $Q - H$ is MN - o.s. in Q . Since h is M_N - Con, $f^{-1}(Q - H)$ is MN - o.s. in U . That is, $U - f^{-1}(H)$ is MN - o.s. in U . Therefore, $f^{-1}(H)$ is MN - c. s. in U . Thus, the inverse image of every MN - c. s. be MN - c. s. Let G be MN - o.s. in Q , Then $Q - G$ is MN - cs in Q . Then $f^{-1}(H - G)$ is M_N -closed in U . That is, $U - f^{-1}$ is M_N -closed in U . Therefore $f^{-1}(G)$ is MN - o.s. in U . Thus, the inverse image of every MN - o.s. in Q is MN - o.s. in U . That is, h is M_N - Con on U .

Theorem 3.8: A function $f: (U, T_R(x)) \rightarrow (Q, T_R(y))$ is M_N - Con $\Leftrightarrow M_N cl(f^{-1}(A)) \subseteq f^{-1}(M_N cl(A))$, $\forall A \subseteq Q$.

Theorem 3.9: A function $f: (U, T_R(x)) \rightarrow (Q, T_R(y))$ is M_N - Con $\Leftrightarrow f^{-1}(M_N int(A)) \subseteq M_N int(f^{-1}(A))$, $\forall A \subseteq Q$.

Proof: let f be M_N - Con and $A \subseteq Q$. Then $M_N int(A)$ is MN - o.s. in $(Q, T_R(y))$. Therefore $f^{-1}(M_N int(A))$ is MN - o.s. in $(U, T_R(x))$. That is $f^{-1}(M_N int(A)) =$

$M_N int[f^{-1}(M_N int(A))]$. Also, $M_N int(A) \subseteq A \Rightarrow f^{-1}(M_N int(A)) \subseteq f^{-1}(A)$. Therefore, $M_N int(f^{-1}(M_N int(A))) \subseteq M_N int(f^{-1}(A))$.

That is $f^{-1}(M_N int(A)) \subseteq M_N int(f^{-1}(A))$. Conversely, let $f^{-1}(M_N int(A)) \subseteq M_N int(f^{-1}(A))$, $\forall A \subseteq Q$. If A is M_N -open in Q , $M_N int(A) = A$. Also $f^{-1}(M_N int(A)) \subseteq M_N int(f^{-1}(A))$. That is $f^{-1}(A) \subseteq M_N int(f^{-1}(A))$. But $M_N int(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $f^{-1}(A) = M_N int(f^{-1}(A))$. Thus, $f^{-1}(A)$ is MN - o.s. in U , \forall MN - o.s. A in Q . Therefore, f is M_N - Con.

Definitions 3.10: Let $(U, T_R(x))$ and $(Q, T_R(H))$ be MN - TS with respect to X and H respectively. A function $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ be called:

- 1- $M_{N\alpha}$ -cont if $f^{-1}(A)$ is $M_{N\alpha}$ -o.s in U , $\forall M_N$ -o.s. $A \subseteq Q$.
- 2- M_{Ns} -cont if $f^{-1}(A)$ is M_{Ns} -o.s. in U , $\forall M_N$ -o.s. $A \subseteq Q$.
- 3- M_{Np} -cont if $f^{-1}(A)$ is M_{Np} -o.s in U , $\forall M_N$ -o.s. $A \subseteq Q$.
- 4- $M_{N\delta}$ -cont if $f^{-1}(A)$ is $M_{N\delta}$ -o.s. in U , $\forall M_N$ -o.s. $A \subseteq Q$.

- 5- $M_{N\beta}$ -cont if $f^{-1}(A)$ is $M_{N\beta}$ -o.s. in U , $\forall M_N$ -o.s. $A \subseteq Q$.
- 6- $M_{N\theta}$ -cont if $f^{-1}(A)$ is $M_{N\theta}$ -o.s. in U , $\forall M_N$ -o.s. $A \subseteq Q$.
- 7- $M_{N\theta s}$ -cont if $f^{-1}(A)$ is $M_{N\theta s}$ -o.s. in U , $\forall M_N$ -o.s. $A \subseteq Q$.

Example 3.11: $U = \{1, 2, 3\}$ with $U/R = \{\{1\}, \{2, 3\}\}$ and $X = \{2, 3\}$. Then $T_R(x) = \{U, \phi, \{2, 3\}\}$, M_N -closed sets are U, ϕ and $\{1\}$, $T_R^\alpha(x) = \{U, \phi, \{2, 3\}\}$ and let $Q = \{a, b, c\}$ with $Q/R = \{\{a\}, \{b, c\}\}$ and $H = \{b, c\}$. Then $T_R(H) = \{Q, \phi, \{b, c\}\}$. Define $f: U \rightarrow Q$ as $f(1) = a, f(2) = b$ and $f(3) = c, f^{-1}(\{b, c\}) = \{2, 3\} \in T_R^\alpha(x)$ and inverse image of \emptyset and Q are ϕ and U respectively. Therefore, f is $M_{N\alpha}$ -cont function.

Example 3.12: $U = \{1, 2, 3\}$ with $U/R = \{\{1\}, \{2, 3\}\}, X = \{1, 2\}$. Then $T_R(x) = \{U, \phi, \{1\}, \{2, 3\}\}$, M_{Ns} (U, x) = $\{U, \phi, \{1\}, \{2, 3\}\}$. Suppose $Q = \{a, b, c\}$ with $Q/R = \{\{a, b\}, \{c\}\}$ and $H = \{b, c\}$. Then $T_R(H) = \{Q, \phi, \{c\}, \{a, b\}\}$. Define $f: U \rightarrow Q$ as $f(1) = c, f(2) = b, f(3) = a$, then $f^{-1}(\{c\}) = \{1\}$ and $f^{-1}(\{a, b\}) = \{2, 3\}$. Hence, f is M_{Ns} - con.

Example 3.13 : $U = \{1, 2, 3\}$ with $U/R = \{\{1\}, \{2, 3\}\}, X = \{2, 3\}$. Then $T_R(x) = \{U, \phi, \{1\}, \{2, 3\}\}$, MN - cs are U, \emptyset and $\{1\}$. M_{Npo} (U, x) = $\{U, \phi, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$ and let $Q = \{a, b, c\}$ with $Q/R = \{\{a\}, \{b, c\}\}$ and $H = \{b, c\}$. Then $T_R(H) = \{Q, \phi, \{b, c\}\}$. Define $f: U \rightarrow Q$ as $f(1) = a, f(2) = b, f(3) = c$, then $f^{-1}(\{b, c\}) = \{2, 3\} \in M_{Npo}(U, x)$, and $f^{-1}(\{\emptyset, Q\}) = \emptyset$ and U respectively. That f is M_{Np} -con.

Example 3.14 : $U = \{1, 2, 3\}$ with $U/R = \{\{1\}, \{2, 3\}\}, X = \{1, 3\}$. Then $T_R(x) = \{U, \phi, \{1\}, \{2, 3\}, M_{Nc}$ - o.s. and $U, \emptyset, \{1\}$ and $\{2, 3\}$. The M_{NR} -o.s. in U are $M_{NR}(U, x) = \{U, \phi, \{1\}, \{2, 3\}\}$ and let $Q = \{p, q, s\}$ with $Q/R = \{\{p\}, \{q, s\}\}$ and $H = \{q, s\}$. Then $T_R(H) = \{Q, \phi, \{q, s\}\}$. Define $f: U \rightarrow Q$ as $f(1) = p, f(2) = q, f(3) = s$, then $f^{-1}(\{q, s\}) = \{2, 3\} \in M_{NR}(U, x)$, and $f^{-1}(\{p\}) = 1 \in M_{NR}(U, x)$. Since $f^{-1}(\{\emptyset, Q\}) = \emptyset$ and U respectively. Therefore f is M_{NR} -cont function.

Example 3.15: $U = \{a, b, c, d\}$ with $U/R = \{\{a, b\}, \{c, d\}\}, X = \{a, b\}$. Then $T_R(x) =$

$\{U, \phi, \{a, b\}\}$, MN – c.s. in U are U, \emptyset and $\{c, d\}$.

$M_{N\beta o}(U, x) =$

$\{U, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{b, c, d\}, \{c, d, a\}, \{d, b\}\}$

and let $Q = \{r, p, q, s\}$ with $Q/R = \{\{r, p\}, \{q, s\}\}$

and $H = \{r, p\}$. Then $T_R(H) = \{Q, \phi, \{r, p\}\}$.

MN – c.s. in Q are Q, \emptyset and $\{q, s\}$. Define $f: U \rightarrow Q$ as $f(a) = r, f(b) = p, f(c) = q, f(d) = s$.

Then $f^{-1}(\{Q, \emptyset\})$ are U and Q . And $f^{-1}(\{r, p\}) = \{a, b\} \in M_{N\beta o}(U, x)$. Hence, f is $M_{N\beta}$ -cont.

Definitions 3.16:[11] A function $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ is called:

1- MN- open map if the image of every MN – o.s. in U is MN – o.s. in Q .

2- MN- closed map if the image of every MN – c.s. in U is MN – c.s. in Q .

Theorem 3.17 : A function $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ is MN – closed map $\Leftrightarrow M_{Ncl}(f(A)) \subseteq f(M_{Ncl}(A)), \forall A \subseteq U$.

Proof: Let f MN- closed, $f(M_{Ncl}(A))$ is MN- closed in Q (because $M_{Ncl}(A)$ is MN – c.s. in Q). Since $A \subseteq M_{Ncl}(A)$ then $f(A) \subseteq f(M_{Ncl}(A))$. Therefore $M_{Ncl}(f(A)) \subseteq f(M_{Ncl}(A))$ if $f(M_{Ncl}(A))$ is MN – c.s. containing $f(A)$.

Conversely, if $M_{Ncl}(f(A)) \subseteq f(M_{Ncl}(A)), \forall A \subseteq U$ and if E is MN – c.s. in U , then $M_{Ncl}(E) = E$, then $f(E) \subseteq f(M_{Ncl}(E)) = f(E)$. Thus, $f(E) \subseteq M_{Ncl}(f(E))$ is MN – c.s. in Q . Then f is a MN- closed map.

Theorem 3.18 : A function $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ is MN – open map $\Leftrightarrow f(M_{Nint}(A)) \subseteq M_{Nint}(f(A)), \forall A \subseteq U$.

Proof is to the of similar to the theorem 3.18.

Definition 3.19 : A function $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ be called a M_N -homomorphism (Hom_{M_N}) if:

1) f is 1 – 1 and onto.

2) f is M_N -continuous.

3) f is M_N -open function.

Theorem 3.20: Let $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ be a bijective mapping . Then f is Hom_{M_N} iff f is MN- closed and M_N -continuous.

Proof. Suppose that f be a Hom_{M_N} , then f is M_N - continuous. For any G is MN – c.s. in U , we get U/G is MN – o.s. and $f(U/G)$ is MN – o.s. in Q . That is, $Q/f(G)$ is MN – o.s. in Q . When $f(G)$ is MN – c.s. in Q (because the image of \forall MN – c.s. in U is MN – c.s. in Q)then f is MN- closed. Conversely, since f is MN- closed and M_N -continuous and K is MN – o.s. in U , then U/K is MN – c.s. in U . Since f is MN – c.s. and $f(U/K) = Q - f(K)$ is MN- closed in Q . Therefore, $f(K)$ is MN- open in Q . Then f is Hom_{M_N} .

Theorem 3.21: Let $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ be a one-one map, then f is a Hom_{M_N} iff $M_{Ncl}(f(A)) = f(M_{Ncl}(A)), \forall A \subseteq U$.

Result 3.22 : 1) M_{NR} -continuous $\Rightarrow M_N$ -continuous.

2) M_N -continuous $\Rightarrow M_{N\alpha}$ -continuous.

3) $M_{N\alpha}$ -continuous $\Rightarrow M_{NS}$ -continuous.

4) $M_{N\alpha}$ -continuous $\Rightarrow M_{Np}$ -continuous.

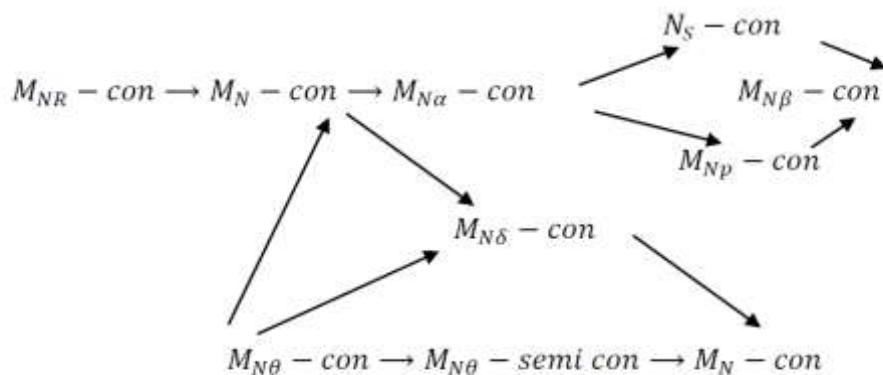
5) M_{NS} -continuous and M_{Np} -continuous $\Rightarrow M_{N\beta}$ -continuous.

6) M_N -continuous $\Rightarrow M_{N\delta}$ -continuous $\Rightarrow M_N$ -continuous.

7) $M_{N\theta}$ -continuous $\Rightarrow M_{N\delta}$ -continuous and $M_{N\theta}$ -semi-continuous and M_N -continuous. 8) $M_{N\theta}$ -continuous $\Rightarrow M_N$ -continuous.

Remark 3.23: the following diagram explains the relations between M_N -continuous functions.

Diagram (2)

The relationship between M_N -continuous functions

References

1. EL-Deeb S.N, AbdEL-MonSef M.E and Mahmoud R.A.,(1983) " p-open sets and β -continuous mapping", Bull. fac. Sci. Assifil Univ. Vol.12. No. 1, pp. 77-90.
2. Levine N.(1963),"Semi-open Sets and Semi continuity in topological space", Amer. Math. Monthly. Vol. 70, No.1, pp. 36-41.
3. Murad, N.A.,and T. H. JaSim,(2020)," N^* open Sets Via Nano topological spaces", Un published master thesis, college of computer sciences and Mathematics, University of Tikrit.
4. Mohammed N. H., and Shihab A. A.,(2022) M-open Nano Topological spaces, , first intenational conference for physics and mathematics IEEE Accepted.
5. Nasef A. A., Aggour A., Darwesh. S.M, (2016), on Some classes of nearly open sets in nano topological spaces, Journal of the Egyptian Mathematical Society, Vol. 24, pp. 585-589.
6. Pankajam V. and K.Kavitha,(2017)," δ -opersets and δ nano continuity in δ nano topological spaces", International Journal of Innovative Science and Research Technology, Vol. 2,12, pp 110 – 118.
7. Refathy A. and Gnanambal Ilango,(2015)"on Nano β -open sets", Int. Jr. of engineering, contemporary mathematics and sciences. Vol. 1, No 2,
8. Richard C.,(2013)"Studies on Nano topological spaces", Ph. D. thesis, Madurai Kamaraj University, India.
9. Sathishmohan P.Rajendran V, Devika A and VaniR, (2017), on nano Semi-continuity and nano pre – continuity, International Journal of Applied Research, Volume 3, No 2, pp. 76-79.
10. Thivagar M., and Carmel Richard,(2013),"on Nano Forms of Weakly open sets", International Journal of Mathematics and Statistics Invention, Vol.1, No.1, pp 31-37.
11. Thivagar M., and Carmel Richard, (2013). On Nano continuity, Mathematical Theory and Modeling. Vol.3, No 7, pp. 32-37.
12. Thivagar M., Saeid iafari ,and Carmel Richard, "Remarks on Weak forms of Nano continuity", e-mail: mlthivagar@yahoo.com.
13. Thivagar M., and Carmel Richard, (2015),"on Nano continuity in a strong form", international journal of pure Applied Mathematics. Vol. 101, No. 5, pp. 893-904.