

On Some Types Nano – M – Continuous Functions

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ABSTRACT

The aim of this work is to definition new type of Nano- M -continuous function. It is named Nano-M-(Nano semi, Nano pre_o, Nano α _o, Nano β _o, Nano δ _o – Nano θ _o, Nano θ _o, Nano Re_o, Nano θ_s _o) continuous function and to reach a relationship with types of Nano- M - cont function are studied with some examples, properties and necessary theorems are studied on it.

Keywords:

Nano-Continuous function , Nano – M – Continuous function

1. Introduction

In 2013 Thivagar M. Lellis [10] idea of Nano-topological space (N_{TS}) with respect to a subset X of universe U which is defined as an upper and lower approximation of x . The element of N_{TS} is called a Nano-open sets (N_{os}).

In 1963 Levine [2] establish the notion of semi-cont. function. In 1983 M.E. Abd EL-Monsef [1] establish the notion of β -cont. function, α -cont function [12], pre-cont. function [9], δ - cont. function, θ – semi cont. function, Regular-cont. function [3], θ – cont. function. In 2022 [4] Mohammed N.H and Shihab A. A and we will intrudes new type of Nano-cont. function and introduced definition of Nano-M - open set.

2. Preliminaries

A subset A of a space (X, T) is called semi-open(se_o) [10] (resp. α -open(α_o) [10], β -open(β_o) [7], pre open(pr_o) [10], δ –

open(δ_o) [6], θ -open(θ_o) [8], Regular-open(Re_o) [10], θ -semi-open(θ_s_o) [6] set. The complement of se_o (resp., α_o , β_o , pr_o , δ_o , θ_o , Re_o , θ_s_o) set is said to be semi-closed(se_c) (resp., α_c , β_c , pre_c , δ_c , θ_c , Regular- c , θ_s_c) set. Intersection of all se_c (resp. α_c , β_c , pre_c , δ_c , θ_c , Regular- c , θ_s_c) sets containing A is called the semi- closure (resp. α - closure, β - closure, pre -closure, δ - closed, θ - closure, Regular- closure, θ - semi - closure) and is denoted by $Scl(A)$ [resp. $\alpha cl(A)$, $\beta cl(A)$, $pcl(A)$, $\theta cl(A)$, $Rcl(A)$, $\delta cl_\theta(A)$].

The union of all se_o (resp. α_o , β_o , pr_o , δ_o , θ_o , Re_o , θ_s_o) sets contained in A is said semi- interior (resp. α - interior, β - interior, pre interior, δ - interior, θ - interior, Regular-interior, θ - semi - interior) and denoted by $Sint(A)$ [resp. $\alpha int(A)$, $\beta int(A)$, $pint(A)$, $\theta int(A)$, $Rint(A)$, $\delta int_\theta(A)$].

The family of all semi-cont (resp., α -cont, β -cont, pre-cont, δ -cont, θ -cont, Regular-cont, θ -semi-cont, α -open set in U , \forall N -open set is denoted by $\delta\text{cont}(x)$, [resp., α -cont(x), β -cont(x), θ -cont(x), R -cont(x), δ -cont(x)].

Definitions 2.1.:[10] Let $(U, T_R(x))$ be a N_TS and $A \subseteq U$. Then A be called.

- 1- N_α -o. if $A \subseteq N_{int}(N_{cl}(N_{int}(A)))$.
- 2- N -pre-o. if $A \subseteq N_{int}(N_{cl}(A))$.
- 3- N_δ -o. if $A \subseteq N_{cl}(N_{int}(A))$.
- 4- $N_{\theta S}$ -o. if $A \subseteq \overline{A_\theta^0}$. [6]
- 5- N_R -o. if $A = N_{int}(N_{cl}(A))$.
- 6- N_θ -o. if $A = NA_\theta^0$. [8]
- 7- N_δ -o. if $A \subseteq \overline{A_\delta^0}$. [6]
- 8- N_β -o. if $A \subseteq N_{cl}(N_{int}(N_{cl}(A)))$. [7]

3. Some Types of M-N-continuity

Definition 3.1:[4] let $(U, T_R(x))$ be N_TS . The subset A of U is called MN-open set (MN - o.s.) in N_TS if $A \subseteq \overline{NA_\theta^0} \cup \overline{NA_\delta^0}$.

Remark 3.2: The complement of MN - o.s. is said to be MN-closed set(MN - c.s.)

Definitions 3.3: Let $(U, T_R(x))$ and $(Q, T_R(H))$ be N_T spaces. A function $f: (U, T_R(x)) \rightarrow (Q, T_R(A))$ is called:

- 1- Nano-continuous($N - Con$) if $f^{-1}(A)$ is N -openset in U , \forall N -open set A in Q . [11]

2- Nano α -continuous ($N_\alpha - Con$) if $f^{-1}(A)$ is N_α -open set in U , \forall N -open set A in Q . [12]

3- N_{pre} - continuous($N - pre - Con$) if $f^{-1}(A)$ is N -pre-o.s. in U , \forall A is N -pre - o.s. in Q . [9]

4- N_β -continuous($N_\beta - Con$) if $f^{-1}(A)$ is N_β - o.s. in U , \forall A is N_β - o.s. in Q . [5]

5- N_S -continuous($N_\delta - Con$) if $f^{-1}(A)$ is N_S - o.s. in U , \forall A is N_S - o.s. in Q . [13]

6- N_R -continuous ($N_R - Con$) if $f^{-1}(A)$ is N_R - o.s. in U , \forall A is N_R - o.s. in Q . [3]

7- N_θ -continuous($N_\theta - Con$) if $f^{-1}(A)$ is N_θ - o.s. in U , \forall A is N_θ - o.s. in Q .

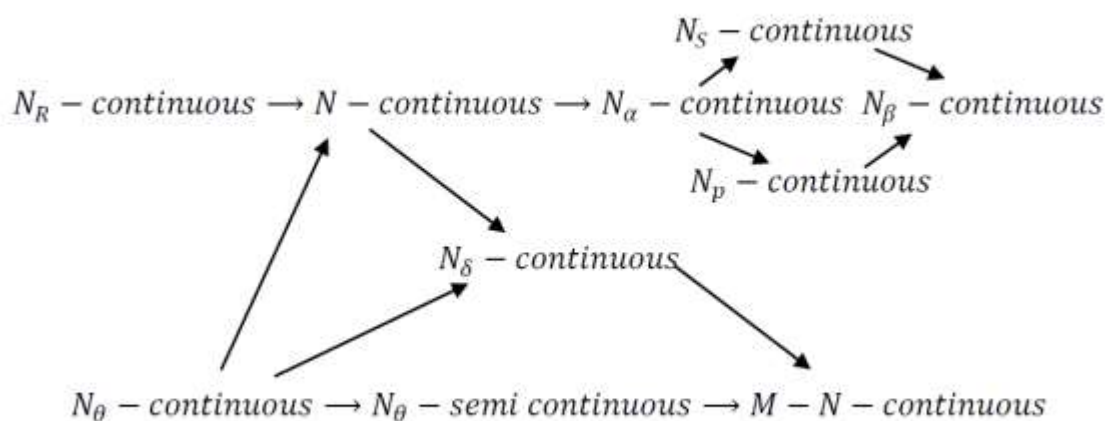
8- N_θ -semi-continuous($N_{\theta S} - Con$) if $f^{-1}(A)$ is $N_{\theta S}$ - o.s. in U , \forall A is $N_{\theta S}$ - o.s. in Q .

9- N_δ -continuous($N_\delta - Con$) if $f^{-1}(A)$ is N_δ - o.s. in U , \forall A is N -open set in Q .

Definition 3.4: let $(U, T_R(x))$ and $(Q, T_R(H))$ be M-Nano-topological space (MN - TS). Then a function $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ is M_N -continuous ($M_N - Con$) on U if $f^{-1}(A)$ is MN - o.s. in U , \forall A is MN - o.s. in Q .

Remark 3.5: The following diagram explains the relations between continuous functions

Diagram (1)



The relationship between continuous functions

Example 3.6: Let $U = \{a, b, c, d\}$ with $T_R(x) = \{U, \phi, \{a, b, c\}\}$, then $MN - o.s. (U) = \{U, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$ and let $Q = \{1, 2, 3, 4\}$ with $T_R(H) = \{Q, \phi, \{1, 2, 3\}\}$, then

$MN - o.s. (Q) = \{Q, \phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$. Define $f: U \rightarrow Q$ as $f(a) = 1, f(b) = 2$ and $f(c) = 3$. Therefore f is M_N -cont. mapping on U , so the above diagram (1) achieves it.

Theorem 3.7: A function $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ is $M_N - \text{Con} \Leftrightarrow f^{-1}(A)$ MN - c.s. in Q is MN - c.s. in U .

Proof: h is $M_N - \text{Con}$, H is MN - c.s. in Q . That is, $Q - H$ is MN - o.s. in Q . Since h is $M_N - \text{Con}$, $f^{-1}(Q - H)$ is MN - o.s. in U . That is, $U - f^{-1}(H)$ is MN - o.s. in U . Therefore, $f^{-1}(H)$ is MN - c.s. in U . Thus, the inverse image of every MN - c.s. be MN - c.s. Let G be MN - o.s. in Q , Then $Q - G$ is MN - cs in Q . Then $f^{-1}(H - G)$ is M_N -closed in U . That is, $U - f^{-1}$ is M_N -closed in U . Therefore $f^{-1}(G)$ is MN - o.s. in U . Thus, the inverse image of every MN - o.s. in Q is MN - o.s. in U . That is, h is $M_N - \text{Con}$ on U .

Theorem 38: A function $f: (U, T_R(x)) \rightarrow (Q, T_R(y))$ is $M_N - \text{Con} \Leftrightarrow M_N \text{cl}(f^{-1}(A)) \subseteq f^{-1}(M_N \text{cl}(A))$, $\forall A \subseteq Q$.

Theorem 3.9: A function $f: (U, T_R(x)) \rightarrow (Q, T_R(y))$ is $M_N - \text{Con} \Leftrightarrow f^{-1}(M_N \text{int}(A)) \subseteq M_N \text{int}(f^{-1}(A))$, $\forall A \subseteq Q$.

Proof: let f be $M_N - \text{Con}$ and $A \subseteq Q$. Then $M_N \text{int}(A)$ is MN - o.s. in $(Q, T_R(y))$. Therefore $f^{-1}(M_N \text{int}(A))$ is MN - o.s. in $(U, T_R(x))$. That is $f^{-1}(M_N \text{int}(A)) = M_N \text{int}[f^{-1}(M_N \text{int}(A))]$. Also, $M_N \text{int}(A) \subseteq A \Rightarrow f^{-1}(M_N \text{int}(A)) \subseteq f^{-1}(A)$. Therefore, $M_N \text{int}(f^{-1}(M_N \text{int}(A))) \subseteq M_N \text{int}(f^{-1}(A))$.

That is $f^{-1}(M_N \text{int}(A)) \subseteq M_N \text{int}(f^{-1}(A))$. Conversely, let $f^{-1}(M_N \text{int}(A)) \subseteq M_N \text{int}(f^{-1}(A))$, $\forall A \subseteq Q$. If A is M_N -open in Q , $M_N \text{int}(A) = A$. Also $f^{-1}(M_N \text{int}(A)) \subseteq M_N \text{int}(f^{-1}(A))$. That is $f^{-1}(A) \subseteq M_N \text{int}(f^{-1}(A))$. But $M_N \text{int}(f^{-1}(A)) \subseteq f^{-1}(A)$. Therefore $f^{-1}(A) = M_N \text{int}(f^{-1}(A))$. Thus, $f^{-1}(A)$ is MN - o.s. in U , \forall MN - o.s. A in Q . Therefore, f is $M_N - \text{Con}$.

Definitions 3.10: Let $(U, T_R(x))$ and $(Q, T_R(H))$ be MN - TS with respect to X and H respectively. A function $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ be called:

- 1- $M_{N\alpha}$ -cont if $f^{-1}(A)$ is $M_{N\alpha}$ -o.s in U , $\forall M_N$ -o.s. $A \subseteq Q$.
- 2- M_{Ns} -cont if $f^{-1}(A)$ is M_{Ns} -o.s. in U , $\forall M_N$ -o.s. $A \subseteq Q$.
- 3- M_{Np} -cont if $f^{-1}(A)$ is M_{Np} -o.s in U , $\forall M_N$ -o.s. $A \subseteq Q$.
- 4- $M_{N\delta}$ -cont if $f^{-1}(A)$ is $M_{N\delta}$ -o.s. in U , $\forall M_N$ -o.s. $A \subseteq Q$.

5- $M_{N\beta}$ -cont if $f^{-1}(A)$ is $M_{N\beta}$ -o.s. in U , $\forall M_N$ -o.s. $A \subseteq Q$.

6- $M_{N\theta}$ -cont if $f^{-1}(A)$ is $M_{N\theta}$ -o.s. in U , $\forall M_N$ -o.s. $A \subseteq Q$.

7- $M_{N\theta s}$ -cont if $f^{-1}(A)$ is $M_{N\theta s}$ -o.s. in U , $\forall M_N$ -o.s. $A \subseteq Q$.

Example 3.11: $U = \{1, 2, 3\}$ with $U/R = \{\{1\}, \{2, 3\}\}$ and $X = \{2, 3\}$. Then $T_R(x) = \{U, \phi, \{2, 3\}\}$, M_N -closed sets are U, ϕ and $\{1\}$, $T_R^\alpha(x) = \{U, \phi, \{2, 3\}\}$ and let $Q = \{a, b, c\}$ with $Q/R = \{\{a\}, \{b, c\}\}$ and $H = \{b, c\}$. Then $T_R(H) = \{Q, \phi, \{b, c\}\}$. Define $f: U \rightarrow Q$ as $f(1) = a, f(2) = b$ and $f(3) = c, f^{-1}(\{b, c\}) = \{2, 3\} \in T_R^\alpha(x)$ and inverse image of ϕ and Q are ϕ and U respectively. Therefore, f is $M_{N\alpha}$ -cont function.

Example 3.12: $U = \{1, 2, 3\}$ with $U/R = \{\{1\}, \{2, 3\}\}$, $X = \{1, 2\}$. Then $T_R(x) = \{U, \phi, \{1\}, \{2, 3\}\}$, $M_{Nso}(U, x) = \{U, \phi, \{1\}, \{2, 3\}\}$. Suppose $Q = \{a, b, c\}$ with $Q/R = \{\{a, b\}, \{c\}\}$ and $H = \{b, c\}$. Then $T_R(H) = \{Q, \phi, \{c\}, \{a, b\}\}$. Define $f: U \rightarrow Q$ as $f(1) = c, f(2) = b, f(3) = a$, then $f^{-1}(\{c\}) = \{1\}$ and $f^{-1}(\{a, b\}) = \{2, 3\}$. Hence, f is M_{Ns} -con.

Example 3.13 : $U = \{1, 2, 3\}$ with $U/R = \{\{1\}, \{2, 3\}\}$, $X = \{2, 3\}$. Then $T_R(x) = \{U, \phi, \{1\}, \{2, 3\}\}$, MN - cs are U, ϕ and $\{1\}$. $M_{Npo}(U, x) = \{U, \phi, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}\}$ and let $Q = \{a, b, c\}$ with $Q/R = \{\{a\}, \{b, c\}\}$ and $H = \{b, c\}$. Then $T_R(H) = \{Q, \phi, \{b, c\}\}$. Define $f: U \rightarrow Q$ as $f(1) = a, f(2) = b, f(3) = c$, then $f^{-1}(\{b, c\}) = \{2, 3\} \in M_{Npo}(U, x)$, and $f^{-1}(\{\phi, Q\}) = \phi$ and U respectively. That f is M_{Np} -con.

Example 3.14 : $U = \{1, 2, 3\}$ with $U/R = \{\{1\}, \{2, 3\}\}$, $X = \{1, 3\}$. Then $T_R(x) = \{U, \phi, \{1\}, \{2, 3\}\}$, M_{Nc} -o.s. and $U, \phi, \{1\}$ and $\{2, 3\}$. The M_{NR} -o.s. in U are $M_{NR}(U, x) = \{U, \phi, \{1\}, \{2, 3\}\}$ and let $Q = \{p, q, s\}$ with $Q/R = \{\{p\}, \{q, s\}\}$ and $H = \{q, s\}$. Then $T_R(H) = \{Q, \phi, \{q, s\}\}$. Define $f: U \rightarrow Q$ as $f(1) = p, f(2) = q, f(3) = s$, then $f^{-1}(\{q, s\}) = \{2, 3\} \in M_{NR}(U, x)$, and $f^{-1}(\{p\}) = 1 \in M_{NR}(U, x)$. Since $f^{-1}(\{\phi, Q\}) = \phi$ and U respectively. Therefore f is M_{NR} -cont function.

Example 3.15: $U = \{a, b, c, d\}$ with $U/R = \{\{a, b\}, \{c, d\}\}$, $X = \{a, b\}$. Then $T_R(x) =$

$\{U, \phi, \{a, b\}\}$, MN – c. s. in U are U, \emptyset and $\{c, d\}$.

$M_{N\beta o}(U, x) =$

$\{U, \phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{b, c, d\}, \{c, d, a\}, \{a, d, b\}\}$

and let $Q = \{r, p, q, s\}$ with $Q/R = \{\{r, p\}, \{q, s\}\}$

and $H = \{r, p\}$. Then $T_R(H) = \{Q, \phi, \{r, p\}\}$.

MN – c. s. in Q are Q, \emptyset and $\{q, s\}$. Define $f: U \rightarrow$

Q as $f(a) = r, f(b) = p, f(c) = q, f(d) = s$.

Then $f^{-1}(\{Q, \emptyset\})$ are U and Q . And $f^{-1}(\{r, p\}) =$

$\{a, b\} \in M_{N\beta o}(U, x)$. Hence, f is $M_{N\beta}$ -cont.

Definitions 3.16:[11] A function $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ is called:

1- MN- open map if the image of every MN – o.s. in U is MN – o.s. in Q .

2- MN- closed map if the image of every MN – c.s. in U is MN – c.s. in Q .

Theorem 3.17 : A function $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ is MN – closed map $\Leftrightarrow M_{Ncl}(f(A)) \subseteq f(M_{Ncl}(A)), \forall A \subseteq U$.

Proof: Let f MN- closed, $f(M_{Ncl}(A))$ is MN- closed in Q (because $M_{Ncl}(A)$ is MN – c.s. in Q).

Since $A \subseteq M_{Ncl}(A)$ then $f(A) \subseteq f(M_{Ncl}(A))$.

Therefore $M_{Ncl}(f(A)) \subseteq f(M_{Ncl}(A))$ if $f(M_{Ncl}(A))$ is MN – c.s. containing $f(A)$.

Conversely, if $M_{Ncl}(f(A)) \subseteq f(M_{Ncl}(A)), \forall A \subseteq U$ and if E is MN – c.s. in U , then $M_{Ncl}(E) = E$, then $f(E) \subseteq f(M_{Ncl}(E)) = f(E)$. Thus, $f(E) \subseteq M_{Ncl}(f(E))$ is MN – c.s. in Q . Then f is a MN- closed map.

Theorem 3.18 : A function $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ is MN – open map $\Leftrightarrow f(M_{Nint}(A)) \subseteq M_{Nint}(f(A)), \forall A \subseteq U$.

Proof is to the of similar to the theorem 3.18.

Definition 3.19 : A function $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ be called a M_N -homomorphism (Hom_{M_N}) if:

1) f is 1 – 1 and onto.

2) f is M_N -continuous.

3) f is M_N -open function.

Theorem 3.20: Let $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ be a bijective mapping. Then f is Hom_{M_N} iff f is MN- closed and M_N -continuous.

Proof. Suppose that f be a Hom_{M_N} , then f is M_N -continuous. For any G is MN – c.s. in U , we get U/G is MN – o.s. and $f(U/G)$ is MN – o.s. in Q . That is, $Q/f(G)$ is MN – o.s. in Q . When $f(G)$ is MN – c.s. in Q (because the image of \forall MN – c.s. in U is MN – c.s. in Q) then f is MN- closed. Conversely, since f is MN- closed and M_N -continuous and K is MN – o.s. in U , then U/K is MN – c.s. in U . Since f is MN – c.s. and $f(U/K) = Q - f(K)$ is MN- closed in Q . Therefore, $f(K)$ is MN- open in Q . Then f is Hom_{M_N} .

Theorem 3.21: Let $f: (U, T_R(x)) \rightarrow (Q, T_R(H))$ be a one-one map, then f is a Hom_{M_N} iff $M_{Ncl}(f(A)) = f(M_{Ncl}(A)), \forall A \subseteq U$.

Result 3.22 : 1) M_{NR} -continuous $\Rightarrow M_N$ -continuous.

2) M_N -continuous $\Rightarrow M_{N\alpha}$ -continuous.

3) $M_{N\alpha}$ -continuous $\Rightarrow M_{NS}$ -continuous.

4) $M_{N\alpha}$ -continuous $\Rightarrow M_{Np}$ -continuous.

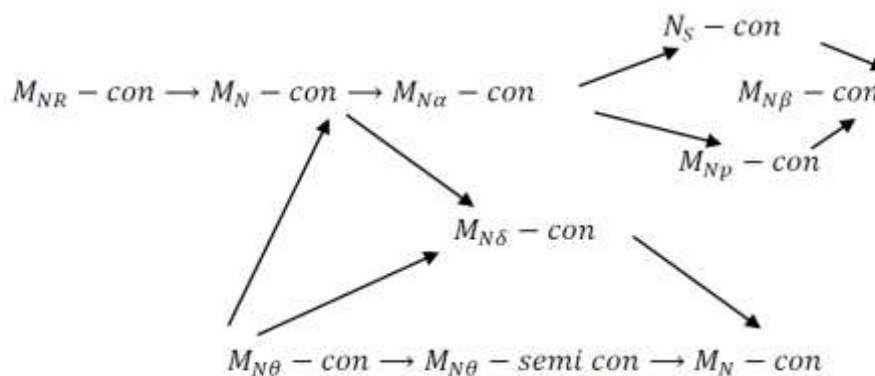
5) M_{NS} -continuous and M_{Np} -continuous $\Rightarrow M_{N\beta}$ -continuous.

6) M_N -continuous $\Rightarrow M_{N\delta}$ -continuous $\Rightarrow M_N$ -continuous.

7) $M_{N\theta}$ -continuous $\Rightarrow M_{N\delta}$ -continuous and $M_{N\theta}$ -semi-continuous and M_N -continuous. 8) $M_{N\theta}$ -continuous $\Rightarrow M_N$ -continuous.

Remark 3.23: the following diagram explains the relations between M_N -continuous functions.

Diagram (2)

The relationship between M_N -continuous functions

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