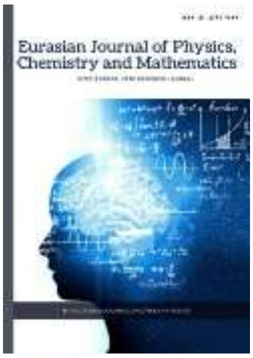


# On Paracompact and Perfectly Zero Dimensional Spaces by Using Semi Feebly Open Sets

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ABSTRACT

The aim of this paper is to introduce contains definitions of paracompact , sf-paracompact , perfectly zero- dimensional spaces , sf-perfectly zero- dimensional spaces, perfect mapping and sf − perfect mapping and we will explain the relationship among them

**Keywords:** paracompact , sf-paracompact , perfectly zero-dimensional , sf-perfectly zero-dimensional , perfect mapping and sf − perfect mapping

## 1.Introduction

The foundation of dimension theory is the "dimension function," It has the properties of d(X)=d(Y) if X and Y are homeomorphic and $d(R^n) = n$ for every positive integer n. It is a function defined on the class of topological spaces where d(X) is an integer or $\infty$ .The dimension functions taking topological spaces to the set {-1,0,1,...} .[1] studied paracompact perfectly zero-dimensional, perfect mapping Actually s-paracompact s-perfectly s-zero-dimensional, perfect mapping were examined using S-open sets in [5], b-paracompact b-perfectly, zero-dimensional, b − perfect mapping , were researched using b-open sets in [6], and f-paracompact, f-perfectly zero-dimensional, f − perfect mapping , were studied using f-open sets in [3], [2] investigated N-paracompact, N-perfectly zero-dimensional, N − perfect mapping utilizing N-open sets. We recall the definitions of paracompact ,perfectly zero-dimensional and perfect mapping [1] , and then use sf − open sets to add sf-paracompact, sf-perfectly zero-dimensionaland sf-perfect mapping .Finally, certain connections between them are investigated, and some conclusions about these notions are established .

## 2.Preliminaries

In this section, we recall some of the basic definitions and theorems.

**Definition(2.1):[1]**

A topological space X is said to be paracompact if each open covering of X has locally finite open refinement.

**Definition(2.2):[1]**

A space X is called perfectly zero dimension space if it has base of open and closed sets and every open covering of X has disjoint open refinement .

**Definition(2.3):[1]**

A continuous surjection $f: X \longrightarrow Y$ is said to be perfect mapping if it is closed and $f^{-1}(y)$ is a compact subset of X for each y in Y.

**Proposition(2.4):[1]**

Each paracmopact regular space is normal and each paracmopact Hausdorff space is $T_4$ −space.

**Proposition(2.5):[1]**

A topological space X is paracmopact and normal space if, and only if, each open cover of X has a locally finite closed refinement.

**Proposition (2.6)[1]:**

A space X is a perfectly zero −dimensional space if, and only if, is paracmopact regular space such that $\dim X = 0$.

**Proposition (2.7):[1]**

Disjoint compact subset in a Hausdorff space have disjoint open neighborhoods

**Definition (2.8): [4]**

Let B be subset of a topological space ,then $B$ called semi feebly open (sf −open) set in X ; if for any semi open set V such that $B \subseteq V$ then $\overline{A}^{f} \subseteq U$. the complement of semi feebly open is called semi feebly closed (sf-closed) that $V \subseteq B^{\circ f}$ where V semi closed set in X.

**Definition (2.9): [4]**

Let X be a topological space, then X is said to be sf-$T_1$-space if for every $x \neq y$ in X there is sf-open sets A and B such that $x \in A, y \notin A$ and $y \in B$, $x \notin B$ .

**Proposition (2.10): [4]**

Let X be a conduct union topological space, then {x} is sf − closed set $\forall x \in X$ iff X is sf -$T_1$-space

**Definition (2.11): [4]**

A space X is called sf-$T_2$-space (sf-Hausdorff space ) if for each $x \neq y$ in X there exists disjoint an sf-open sets U, V such that $x \in U, y \in V$ .

**Definition (2.12): [4]**

A topological space X is said to be sf$\grave{}$-regular space if for each x in X and sf-closed subset A such that $x \notin A$ there exists disjoint sets U, V such that U open set , V is sf-open sets $x \in U, A \subseteq V$.

**Definition (2.13):**[7]

Let X be a topological space, then X is called sf″-regular space if any x in X and sf-closed subset F such that $x \notin F$ there is disjoint sf-open sets A , B such that $x \in A, F \subseteq B$.

**Proposition (2.14):**[4]

Let X be a conduct union topological space, then X is sf$\grave{}$-normal space iff ∀ sf -closed set $E \subseteq X$ , and ∀ sf -open set V in X ∋ $E \subseteq V$ , ∃ sf -open set U ∋ $E \subseteq U \subseteq \overline{U}^{sf} \subseteq V$

**Definition(2.15):**

A topological space X is said to be sf″-normal space if for any disjoint sf -closed set $N_1$ , $N_2$ , there exists is disjoint sets $V_1$ , $V_2$ such that $V_1$ sf − open , $V_2$ open set and $N_1 \subseteq V_1$ , $N_2 \subseteq V_2$ .

**Definition(2.16):[4]**

Let $f: X \longrightarrow Y$ be a function of a topological space (X,τ) into a topological space (Y,τ$\grave{}$),then f is called an sf*-continuous function if $f^{-1}(A)$ is an open set in X for every sf-open set A in Y.

**Definition(2.17):[4]**

A function $f: (X, \tau) \longrightarrow (Y, \tau\grave{})$ is called sf-open function if f(A)
is an sf-open set in Y for every open set A in X .

**Definition(2.18):[4]**

f is called sf − closed function if f(F) is sf-closed set in Y for every closed set F in X.

**Definition (2.19): [4]**

Let X be a topological space and $A \subseteq X$. An sf-neighborhood of A is every subset of X which contains an sf-open set containing A . The sf- neighborhood of a subset {x} is said to be sf- neighborhood of the point x .

**Definition (1.20): [4]**

The a family $\{A_\lambda : \lambda \in \Lambda\}$ of subsets of a topological space $X$ is said to be $sf$-locally finite if for each point $x$ of $X$ there exists an $sf$-neighborhood $N_x$ of $x$ such that the set $\{\lambda \in \Lambda : N_x \cap A_\lambda \neq \emptyset\}$ is finite.

**Definition(2.21):[4]**

A topological space X is said to be sf − paracmopact if each sf − open covering of X, has sf − locallyfinite sf − open refinement.

**Definition (2.22):[7]**

A non-empty collection $\mathfrak{B}_{(x)}$ of sf-neighborhoods for $x \in X$ is called sf-base for sf-neighborhood system of x of all sf-open sets in X if and only if for every sf-neighborhood $N_x$ of x there is $B \in \mathfrak{B}_{(x)}$ such that $B \subset N_x$

## 3.The Main Results

**Definition(3.1):**

A topological space X is said to be $sf^*-$ paracmopact if each $sf-$ open covering of X, has locallyfinite open refinement.

**Definition(3.2):**

$sf^*-$continuous surjection $f: X \longrightarrow Y$ is said to be $sf-$ perfectly mapping if and only if it is $sf-$ closed and $f^{-1}(y)$ is a compact subset of X for each y in Y.

**Definition(3.3):**

A space X is called $sf-$ perfectly zero dimension space if and only if it has $sf-$ base of $sf-$ open and $sf-$ closed sets and every $sf-$ open covering of X has disjoint $sf-$ open refinement .

**Proposition (3.4):**

Let X be $sf^*-$ paracmopact subset of X, let A be a subset of X and let B be a $sf-$ closed sub set of X which disjoint from A. If every $x \in B$ there exist disjoint $sf-$ open sets $U_x$ and $V_x$ such that $A \subseteq U_x$ and $x \in V_x$ , then there exists sets U, V such that U is $sf-$ open and V is open and $A \subseteq U, B \subseteq V$.

**Proof:**

The $sf-$ open covering of $sf^*-$ paracmopact space X which consist of X/B together with the sets $V_x$ for x in B has a locally finite open refinement $\{W_\gamma\}_{\gamma\in\Gamma}$. Let $\Gamma = \{\gamma \in \Gamma: W_\gamma \subseteq V_\gamma$ for some x in B} , if $\gamma \in \Gamma$ then $U_x \cap W_\gamma = \emptyset$ for some x ,so that$A \cap W_\gamma = \emptyset$ . Now let $U = X/\bigcup_{\gamma\in\Gamma} W_\gamma$ and $V = \bigcup W_\gamma$, then $A \subseteq U$ and $B \subseteq V$ and U,V are disjoint sets. Clearly U is $sf-$ open set and V is open set

**Theorem (3.5):**

if X is $sf^*-$ paracmopact sf-Hausdorff space ,then X is $sf^{`}-$ regular space

**proof:**

let $x \in X$ and B be sf-closed set in X such that $x \notin B$ then for any $y \in B$,there is disjoint sf-open set $U_y$ and $V_y$ where $x \in U_y$ , $y \in V_y$ (since X is sf-Hausdorff space).Then by proposition (3.4) there is disjoint sets U and V such that U is sf-open set and V is open set and $x \in U, B \subseteq V$ .therefor X is $sf^{`}-$ regular

**Proposition(3.6):**

Each $sf^*-$ paracmopact $sf^{``}-$ regular topological space is $sf^{``}-$ normal space.

**Proof:**

Let X be $sf^*-$ paracmopact $sf^{``}-$ regular and let A and F be disjoint $sf-$ closed sets in X. Since A is $sf-$ closed set of the $sf^{``}-$ regular topological space X. Hence for every x in F there exist disjoint $sf-$ open sets $U_x$ and $V_x$ such that $A \subseteq U_x, x \in V_x$. It follows from Proposition(3.4) that there exist disjoint sets U and V such that U $sf-$ open set and V open set and $A \subseteq U, F \subseteq V$. Thus X is $sf^{``}-$ normal space .

**Theorem (3.7):**

If each finite $sf-$ open covering of a space X has a sf-locally finite sf-closed refinement , then X is $sf^{`}-$ normal space .

**Proof:**

Let X be a topological space each finite sf-open covering which has a locally finite sf - closed refinement and Let A,B be disjoint sf - closed sets of X. The sf - open covering { X / A , X / B } of X hast sf-locally finite sf - closed refinement F. Let U be the union of members of F disjoint from A and V be the union of the members of F disjoint from B. Then U and B are sf - closed sets and U∪V = X . Thus if G = x / U and W = X / V , then G , W are disjoint sf - open sets such that A⊆G , B⊆W . Hence X is $sf^{`}$- normal space .

**Theorem(3.8):**

Let $\{A_\alpha\}_{\alpha\in\Lambda}$ be a family of subsets of a space X , let $\{B_\gamma\}_{\gamma\in\Gamma}$be sf-locally finite sf-closed covering of X such that for each γ in Γ , the set $B_\gamma \cap \{ \alpha \in \Lambda : B_\gamma \cap A_\alpha = \emptyset\}$ is finite . Then There exists a sf-locally finite family $\{H_\alpha\}_{\alpha\in\Lambda}$ of sf - open sets of X such that $A_\alpha \subseteq H_\alpha$for each α in Λ .

**Proof :**

For each α , let $H_\alpha = X / U \{ B_\gamma / A_\alpha = \emptyset\}$ . Clearly $A_\alpha \subseteq H_\alpha$ , and since $\{B_\gamma\}_{\gamma\in\Gamma}$is sf-locally finite sf-closed , then $H_\alpha$ is sf - open . Let X be point of X , there exists a sf-neighborhood N of x , and a finite subset K of Γ such that $N \cap B_\gamma = \emptyset$ for $\gamma \notin K$ , Hence $N \subseteq \bigcup_{\gamma\in K} B_\gamma$ . Now $H_\alpha \cap B_\gamma \neq \emptyset$ if and only if $A_\alpha \cap B_\gamma \neq \emptyset$ . For each γ in K the set $\{ \alpha \in \Lambda: A_\alpha \cap B_\gamma \neq \emptyset \}$ is finite . Hence the set $\{\alpha \in \Lambda: N \cap H_\alpha \neq \emptyset\}$ is finite

**Theorem(3.9):**

Let X be a topological space. If each sf−open cover of X has a sf−locally finite sf−closed refinement, then X is sf−paracompact sf` −normal

**Proof** :

Let U be an sf−open covering of X and Let $\{F_\lambda\}_{\lambda\in\Lambda}$ be a sf−locally finite sf−closed refinement of U. since $\{F_\lambda\}_{\lambda\in\Lambda}$ is sf−locally finite, each point of X has a sf−neighborhood $G_x$ such that $\{\lambda \in \Lambda : G_x \cap F_\lambda \neq \emptyset\}$ is finite .If $\{E_\lambda\}_{\lambda\in\Gamma}$ is a sf−locally finite sf− closed refinement of the sf−open covering $\{G_x\}_{x\in X}$ of X then for each $\lambda \in \Gamma$ the set $\{\lambda \in \Lambda : E_\lambda \cap F_\lambda \neq \emptyset\}$ is finite . It follows from Theorem (3.8) that there exists a sf−locally finite family $\{V_\lambda\}_{\lambda\in\Lambda}$ of sf−open sets, such that $F_\lambda \subseteq V_\lambda$ for each λ . for each λ in Λ , Let $U_\lambda$ be member of U such that $F_\lambda \subseteq U_\lambda$ ,then $\{V_\lambda \cap \mathcal{U}_\lambda\}_{\lambda\in\Lambda}$ is a sf−locally finite sf−open refinement of U . Thus X is sf−paracompact . so that by Theorem (3.7) X is sf` −normal space.

**Definition(3.10):**

Let X be a topological space,The sf−covering dimension,sf− dimX, of X is the least integer n where each finite sf-open covering of X has an sf− open refinement of order ≤ n or is ∞ if no such integer exists .Thus sf− dimX = −1 if and only if X is empty,and sf− dimX ≤ n if each finite sf-open covering of X has sf− open refinement of order ≤ n. We have sf− dimX = n if it is true that sf− dimX ≤ n but sf− dimX ≤ n − 1 is not true . Finally sf− dimX = ∞ if for every integer n it is false that sf− dimX ≤ n.

**Theorem(3.11):**

Let X be a topological space . If X has sf-base of sets which are both sf-open and sf-closed , then sf− dim X = 0.

For sf-$T_1$-space the convers is true

**proof:**

let X has a sf-base of sets which are both sf− open and sf− closed

let $\{U_i\}_{i=1}^k$ be a finite sf− open covering of X . It has sf-open refinement $\mathcal{H}$ . If $H \in \mathcal{H}$ then $H \subseteq U_i$ for some i . Let each H in $\mathcal{H}$ associated with one of the sets $U_i$ containing it , and let $V_i$ be the union of those members of $\mathcal{H}$ .That is associated with $U_i$ . thus $V_i$ is sf-open set and hence $\{V_i\}_{i=1}^k$ forms a disjoint open refinement of $\{U_i\}_{i=1}^k$ .Then sf − dim X = 0

conversely:

let X be a sf-$T_1$-space where sf-dim X =0 , if x ∈ X and A is sf-open in X such that x ∈ A , then {A,X-{x}} is finite sf-open cover of X .since sf-dim X=0 then there is sf-open refinement {W,V} of order zero, where W∩V=∅ , W∪V=X ,V⊆ A and W⊆ X − {x} .then V and W are open and closed sets in X , therefor V and W are sf-open and sf-closed such that x ∈ $W^c$ =V⊆ A. Thus X has sf-base of sf-open and sf-closed sets

**Theorem (3.12):**

If X is sf− perfectly zero dimension space then , X is sf − paracmopact space and dim X=0

**Proof:**

Let X be sf− perfectly zero dimension space then X has a base of sf− open and sf− closed sets . Hence by theorem (3.11) sf− dim X = 0 . And since every sf− open covering has disjoint sf− open refinement , then it is sf− locally finite sf− open refinement . Thus X is sf− paracompact space .

**Proposition(3.13)** :

If a mapping $g : x \to y$ is sf−closed $sf^*$ −continuous surjection, then for each y ∈ Y and an open set G in X, where $g^{-1}(y) \subseteq G$, there exists an sf−open set V in Y such that y ∈ V and $g^{-1}(V) \subseteq G$ .

**Proof**:

Let y ∈ Y and G an open in X such that $g^{-1}(y) \subseteq G$ and

let V = Y/g(X/G) ,since G is an open set in X, then X / G is closed in X, g(X/G) sf−closed in Y and hence V = Y/g(X/G) is sf−open in Y .

Now to prove: (1) y ∈ V , (2) $g^{-1}(V) \subseteq G$

(1) since $g^{-1}(y) \subseteq G$, so $X/G \subseteq X/g^{-1}(y) = g^{-1}(Y/y)$ which implies that $g(X/G) \subseteq g(g^{-1}(Y/y)) = (Y/y)$ and hence y ∈ Y/g(X/G) = V

(2) $g^{-1}(V) = g^{-1}[Y/g(X/G)] = X/g^{-1}(g(X/G)) = X/X/G = G$ .

$= X/g^{-1}(g(X/G))$

$\subseteq X/X/G = G$ .

**Proposition(3.14)** :

If a mapping $g: x \to y$ is sf −closed sf* −continuous surjection, then for each $A \subseteq Y$ and each open set G in X, where $g^{-1}(A) \subseteq G$, there exists an sf −open set V in Y such that $A \subseteq V$ and $g^{-1}(V) \subseteq G$.

**Proof**:

Let $A \subseteq Y$ and G an open in X such that $g^{-1}(A) \subseteq G$ and let $V = Y/g(X/G)$ ,since G is an open set in X, then X / G is closed in X, $g(X/G)$ sf −closed in Y and hence $V = Y/g(X/G)$ is sf −open in Y.

Now to prove: (1) $A \subseteq V$ , (2) $g^{-1}(V) \subseteq G$

(1) since $g^{-1}(A) \subseteq G$, so $X/G \subseteq X/g^{-1}(A) = g^{-1}(Y/A)$ which implies that $g(X/G) \subseteq g(g^{-1}(Y/A)) = (Y/A)$ and hence $A \subseteq Y/g(X/G) = V$

(2) $g^{-1}(V) = g^{-1}[Y/g(X/G)]$
$= X/g^{-1}(g(X/G))$
$\subseteq X/X/G = G$.

**Proposition(3.15):**

Let $g: X \to Y$ be a sf −perfect mapping and X is $T_2$−space then Y is sf −$T_2$−space.

**Proof** :

Let a , b be distinct point of Y. then $g^{-1}(a) \cap g^{-1}(b) = \emptyset$ and since g is sf −perfect mapping , then $g^{-1}(a), g^{-1}(b)$ are compact space subsets of $T_2$-space X. hence by proposition (2.7) there exist disjoint open sets V and W such that $g^{-1}(a) \subseteq V$ and $g^{-1}(b) \subseteq W$. Then by proposition (3.13) there exist sf −open sets G, H in Y such that $a \in G$, , $b \in H$ , $g^{-1}(G) \subseteq V$ , $g^{-1}(H) \subseteq W$.It is clear that $g^{-1}(G \cap H) = \emptyset$ then $G \cap H = \emptyset$. And hence Y is sf −$T_2$ space .

**Theorem (3.16):**

Let $(X, \tau)$ and $(Y, \tau^{`})$ be a topological space. A function $g: X \longrightarrow Y$ is sf*-continuous if and only if the invers image under g of every sf-closed in Y is closed in X.

**Proof:**

Assume that f is sf*-continuous and let F be any sf-closed set in Y. To show that $f^{-1}(F)$ is closed set in X. Since g is sf*-continuous and $Y - F$ is sf-open in Y , that $f^{-1}(Y - F) = X - f^{-1}(F)$ is open in X, that is $f^{-1}(F)$ is closed in X. Conversely, let $f^{-1}(F)$ is closed in X for every sf-closed set F in Y . We want to show that f is a sf*-continuous function. Let G be any sf-open in Y, then $Y - G$ is sf-closed in Y and by hypothesis $f^{-1}(Y - G) = X - f^{-1}(G)$ is closed in X , that is $f^{-1}(G)$ is open in X. Hence f be a sf*-continuous .

**Proposition (3.17):**

Let $g: X \to Y$ be a sf −perfect mapping. If X is regular space then y is sf" −regular space .

**Proof :**

Let $y \in Y$ and F be sf −closed subset of Y such that $y \in Y / F$ then by Proposition (3.16) $g^{-1}(F)$ is closed set in Y and $g^{-1}(y)$ is non emply compact sup set in X. clearly, $g^{-1}(y) \cap g^{-1}(F) = \emptyset$ . Let $x \in g^{-1}(y)$ so $x \notin g^{-1}(F)$ since X is regular space, there exist disjoint open set $U_x$. $V_x$ in X such that $x \in U_x$ , $g^{-1}(F) \subseteq V_x$ ,therefore $g^{-1}(y) \subseteq \bigcup_{x \in g^{-1}(y)} U_x$ and then there is $x_1, \dots, x_n \in g^{-1}(y)$ such that $g^{-1}(y) \subseteq U_{x_1} \cup U_{x_2} \cup U_{x_3} \cup \dots \cup U_{x_n}$. Let $U = U_{x_1} \cup U_{x_2} \cup U_{x_3} \cup \dots \cup U_{x_n}$ and let $V = \bigcap_{i=1}^{n} V_{x_i}$ ,then U, V are open sets in X . since $g^{-1}(F) \subseteq V_{x_i}$ for each $i = 1, \dots, n$. then $g^{-1}(F) \subseteq \bigcap_{i=1}^{n} V_i = V$, thus $U_{x_i} \cap V_{x_i} = \emptyset$, for each $i = 1, \dots, n$.

So that $U_{x_i} \cap \left(\bigcap_{i=1}^{n} V_{x_i}\right) = \emptyset$, then $U_{x_i} \cap V = \emptyset$, $i = 1, \dots, n$ thus $U \cap V = \left(\bigcup_{i=1}^{n} U_{x_i}\right) \cap V = \bigcup_{i=1}^{n} \left(U_{x_i} \cap V\right) = \emptyset \cup \emptyset \cup \emptyset \cup \dots \cup \emptyset = \emptyset$,

then there exist disjoint open set U, V such that $g^{-1}(y) \subseteq U$ , $g^{-1}(F) \subseteq V$. Since g is sf −closed ,then by proposition (3.13),(3.14). there exists sf −open sets W, Gin Y such that $y \in W$ , $F \subseteq G$ and $g^{-1}(W) \subseteq U$, $g^{-1}(G) \subseteq V$, $g^{-1}(W) \cap g^{-1}(G) \subseteq U \cap V = \emptyset$,so that $g^{-1}(W \cap G) = \emptyset$. Thus $W \cap G = \emptyset$ and hence Y is sf" −regular space .

**Proposition(3.18)**

If $f: X \to Y$ is sf*-continuous sf-closed surjection mapping and X is normal space ,Then y is sf̀ -normal space.

**Proof:**

Let A,B be disjoint sf − closed in y , thus $g^{-1}(A)$and $g^{-1}(B)$ disjoint closed in X by Proposition (3.16).Since X is normal space then there exist disjoint open sets U, V such that $g^{-1}(A) \subseteq U, g^{-1}(B) \subseteq V$ Now by proposition (3.14) there exist sf − open sets G,H in Y such that $A \subseteq G$ and $g^{-1}(G) \subseteq U$ so that $B \subseteq H$ , $g^{-1}(H) \subseteq V$ .It is clear that $g^{-1}(G) \cap g^{-1}(H) \subseteq U \cap V = \emptyset$ , so that $g^{-1}(G \cap H) = \emptyset$ . and hence $G \cap H = \emptyset$,Then y is sf̀-normal space.

**Theorem(3.19)**

Let X be a paracompact normal space and $g: X \to Y$ is sf-closed $sf^*$-continuous surjection mapping where Y has $sf-$locally finite covering , then Y is $sf-$paracompact $s\hat{f}$ $-$normal space.

**Proof:**

Let $\{G_\lambda\}_{\lambda\in\Lambda}$ be an $sf-$open cover of Y , then $\{ g^{-1}(G_\lambda)\}_{\lambda\in\Lambda}$ is open cover of X. Since X is paracompact normal space then by proposition(2.5) it has is locally finite closed refinement $\{U_\lambda\}_{\lambda\in\Lambda}$ ,Such that $U_\lambda \subseteq g^{-1}(G_\lambda)$ and hence $g(U_\lambda) \subseteq G_\lambda$ ,because Y has $sf-$ locally finite covering then $\{ g(U_\lambda)\}_{\lambda\in\Lambda}$ is sf-locally finite sf- closed refinement of $\{G_\lambda\}_{\lambda\in\Lambda}$ . Then by proposition(3.9) Y is sf -paracompact $s\hat{f}$ -normal space.

**Lemma(3.20):**

Let X be a paracompact Hausddorff space and $g: X \to Y$ is a sf-closed $sf^*$-continuous surjection mapping where Y has $sf-$ locally finite covering then y is $sf-$ paracompact $s\hat{f}$ $-$normal space

**Proof:**

Since X is paracompact Hausddorff space ,then by proposition (2.4) X is normal space . Hence by theorem(3.19) Y is $sf-$ paracompact $s\hat{f}$ $-$normal space

**Corollary(3.21):**

Let X be a perfectly zero-dimension space and $g: X \to Y$ is a sf-closed $sf^*$-continuous surjection mapping where Y has $sf-$ locally finite covering then y is $sf-$ paracompact $s\hat{f}$ $-$normal space.

**Proof:**

Since X is perfectly zero-dimension space ,then by Proposition(2.6) X is paracompact regular and dim X=0,hence by Proposition(2.4) X is normal space.thus by theorem (3.19) Y is $sf-$paracompact $s\hat{f}$ $-$normal space.

**Corollary(3.22):**

Let X be a paracompact Hauddorff space and $g: X \to Y$ is a $sf-$ perfect mapping where Y has $sf-$locally finite covering then y is $sf-$ paracompact $s\hat{f}$ $-$normal $sf-T_2-$space and $sf''-$ regular space

**Proof:**

By Lemma(3.20) Y is $sf-$paracompact $s\hat{f}$ $-$normal space . Since X is $T_2$-space then by proposition (3.15) Y is $sf-T_2-$space and since X is paracompact $T_2$-space , hence X is regular by proposition (2.4) .Then (3.17) Y is $sf''-$ regular space

**Corollary(3.23):**

Let X be a perfectly zero-dimension space and $g: X \to Y$ is a $sf-$ perfect mapping where Y has $sf-$locally finite covering then y is $sf-$ paracompact $s\hat{f}$ $-$normal and $sf''-$regular space

**Proof:**

since X be a perfectly zero-dimension space , then X is paracompact regular space such that dim X=0 by proposition(2.6) .So that X is normal space by proposition(2.4) . Hence by theorem (3.19) Y is $sf-$paracompact $s\hat{f}$ $-$normal .since X is regular space then by proposition (3.17) Y is $sf''-$ regular and $g: X \to Y$ is a $sf-$ perfect mapping where Y has $sf-$ locally finite covering then y is $sf-$ paracompact $s\hat{f}$ $-$normal and $sf''-$regular space

**Theorem(3.24):**

Let X be a paracompact space and $f: X \to Y$ is sf-open $sf^*$-continuous surjection mapping where Y has sf-locally finite covering , then y is sf-paracompact.

**Proof:**

Let $\{G_\lambda\}_{\lambda\in\Lambda}$ be an sf-open covering of Y, then $\{ g^{-1}(G_\lambda)\}_{\lambda\in\Lambda}$ is open covering of X. Since X is paracompact space then there is locally finite open refinement $\{U_\lambda\}_{\lambda\in\Lambda}$ ,Such that $U_\lambda \subseteq g^{-1}(G_\lambda)$ for each $\lambda$ and $\cup_{\lambda\in\Lambda} U_\lambda = X$, then $g(U_\lambda) \subseteq G_\lambda$ and hence $\{ g(U_\lambda)\}_{\lambda\in\Lambda}$ is sf- locally finite sf- open refinement of $\{G_\lambda\}_{\lambda\in\Lambda}$ and covering of Y , then Y is sf-paracompact space.

**Corollary(3.25):**

Let X be a perfectly zero dimensional space and $g: X \to Y$ is a sf-open $sf^*$-continuous surjection mapping where Y has sf-locally finite covering , then y is sf-paracompact space.

**Proof:**

Since X is perfectly zero dimensional space, then by theorem (2.6) X is paracompact space. Thus by theorem (3.24) Y is sf-paracompact space.

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