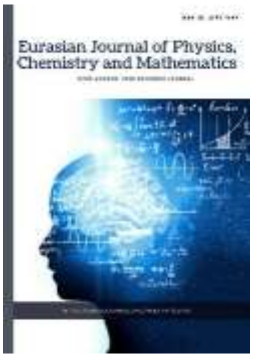

About One Difference Scheme of High Order Accuracy on A Special Uneven Grid

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ABSTRACT

This article deals with the construction and study of exact schemes, as well as their corresponding truncated schemes of the "m" rank for the marginal problem in the case of a system of ordinary second-order differential equations with degeneration.

In this case, the matrix coefficients are considered to be non-self-adjoint, satisfying the Gelder condition on the segment [-1; 1] with a degree μ $(0 < \mu \leq 1)$. Due to the choice of a special uneven grid that takes into account the type of degeneration, a high accuracy of truncated difference schemes of the m-th rank was achieved.

It is proved that when the matrix coefficients P(x), Q(x) satisfy the Gelder condition with a degree μ $(0 < \mu \leq 1)$ on the segment [-1;1] and their elements are piecewise continuous functions on said segment, truncated schemes of the "m" rank have an accuracy of O(). $N^{-(2m+2\mu)}$

This score is not improvable in the class of piecewise continuous functions.

Keywords: Edge problem, degeneracy, exact three-point difference scheme, truncated difference scheme, scheme rank, template, grid, uneven grid, norm, special norm, difference norm, Gelder condition, continuity, grid step, grid node, ordinary differential equation, convergence, Kronecker symbol, template matrix functions, operator, Green function, recurrence relation, matrix series, Lipschitz condition, Cauchy-Buniakovsky inequality, difference scheme error, scalar product of vectors in R^n , energy inequality.

Precise difference schemes for boundary problems were constructed and investigated by A. N. Tikhonov and A. A. Samarsky in [1,2]. With the help of precise schemes, truncated difference schemes of any order of accuracy are built, which play an important role in practical calculations. In [3], these results are transferred to the edge problem in the case of a system of second-order differential equations without degeneration. Precise and truncated difference schemes of any order of accuracy for a non-self-adjoint boundary value problem in the case of a degenerate system of second-order differential equations are considered in the work [5].

In the present work, selecting a special uneven grid on [-1,1] for the type of degeneracy, the above-mentioned results extend to the non-self-adjoint boundary problem in the case of a system of ordinary second-order differential equations with degeneration. As in the scalar case, the construction of such difference schemes relies significantly on the properties of template functions, which are solutions to the corresponding Cauchy problems for a homogeneous system of equations. The paper presents the conditions under which the exact difference scheme can be reduced to a homogeneous divergent form.

1. **Problem statement.** Let's consider the regional task

$L^{(P,Q)}\vec{u}(x) \equiv \frac{d}{dx}$ (P(x))-Q(x)=-(x) , -1<x<1 $\frac{d\vec{u}}{dx}$ $\vec{u}$(x)$\vec{f}$**(1.1)**

P(x)= P(x)=. $\frac{d\vec{u}}{dx}\Big|_{x=-1}$ $\frac{d\vec{u}}{dx}\Big|_{x=+1}\vec{0}$**(1.2)**

Здесь P(x)=((1-x ^{2})P$_1$(x) , P$_1$(x)= j=1, $\|P_{i,j}(x)\|_{i,j=1}^{n}$

Q$_1$(x)= - заданные вещественные, несимметричные $\|q_{i,j}(x)\|_{i,j=1}^{n}$

square matrices of order nxn, ($\vec{u}$(x) $-$ искомая. $\vec{f}$x) are given n dimensional vectors of the function. Matrices satisfy the conditions P$_1(x)$и $Q(x)$

0<$C_1(\vec{y},\vec{y}) \leq$ (P$_1$(x) $\vec{y},\vec{y}) \leq$ $C_2(\vec{y},\vec{y})$,

0< Q(x)$C_3(\vec{y},\vec{y}) \leq$ ($\vec{y},\vec{y}) \leq$ $C_4(\vec{y},\vec{y})$, **(1,3)**

$\forall$ $\vec{y} \in E^n$,, >0 , = (0)$\|\vec{y}\| \neq 0 C_i i\overline{1,4}$

Кроме того, потребуем, что P $_{i,j}$(x) , q $_{i,j}$(x) , f$_i$(x) , $\in Q^{(0)}[-1,1]$**(1.4)**

i.e. the elements of the matrices P$_1$(x), Q(x) and vector ($\vec{f}$x) are piecewise continuous on$[-1; 1]$the function.

Let =<<... <<<... <$\omega_{h_i}\{-1 = x_{-N}x_{-N+1}x_0x_1x_N$=1} uneven on [-1; 1] grid. The nodes and grid pitch are determined by the formulas:

h_i=-=(1-(-0.5)/$x_ix_{i-1}\frac{2}{N}$ iN),

$x_i = sign(x_i)\sum_{p=1}^{i} h_i = sign(x_i)$ $\frac{2i}{N}$(1-i/2N)), i=-N,N. **(1.5)**

Определим шаблонные матричные функции V_j^i(x) , j=1,2; i=$\overline{-N+1, N-1}$,

как Solutions to Cauchy's problems:

$L^{(P,Q)}$ V_j^i(x)=Θ , x_{i-1}<x< , j=1,2 ,x_{i-1}

V_1^i()=E , P(x)$x_{i-1}\delta_{i,-N+1}$ $\frac{dV_j^i(x)}{dx}\Big|_{x=x_{i-1}}$=(1-)E , $\delta_{i,-N+1}$**(1.6)**

V_2^i()=E, P(x)$x_{i+1}\delta_{i,N-1}$ $\frac{dV_2^i(x)}{dx}\Big|_{x=x_{i+1}}$=(1-)E,$\delta_{i,-N-1}$ i.$\overline{-N+1, N-1}$.

где Θ $-$ матрица с нулевыми элементами, E $-$ единичная матрица , $\delta_{i,j}$- The Kronoker symbol.

Lemma 1. Let the conditions (1.3) be satisfied, then the template matrix functions (V_j^ix) , j=1.2; i = meet $\overline{-N+1, N-1}$ the following conditions:

1) (V_j^ix)-linearly independent;

b) (V_j^ix) nevыbirthnыe, i.e.

det(x)≠0, xV_j^i $_{i-1}$<x<x$_{i+1}$ **(1.7)**

The lemma is proved as in the case of a uniform grid (see,[5]).

Lemma 2. Let the conditions (1,3) and P(x)= Then the template matrix functions ($P^*(0), Q(x) = Q^*(x). V_j^i$x), j=1,2, have the following properties:

1) $V_1^i(x_{i+1})$= $V_2^{i*}(x_{i-1})$, i =$\overline{-N+2, N-2}$,
2) $V_1^i(x_{i+1})$= $V_2^{i*}(x_i)$, i =$\overline{-N+2, N-2}$,
3) (1-) $\delta_{i,N-1}$ V_1 (x_{i+1})+ $\delta_{i,N-1}V_2^{i*}$ (x_{i-1}) + (1-) $\delta_{i,-N+1}$ V_2 (x_i)+ V_1^{i*} (dt+ x_i) $\int_{x_{i-1}}^{x_i} Q(t)\, V_1^i(t)$ (1-$\delta_{i,N-1}$)$V_1^i(x_i)$+ V_2^{i*}(dt , =x_i) $\int_{x_i}^{x_{i+1}} Q(t)\, V_2^i(t) i\overline{-N+1, N-1}$.

The proof is carried out, as in [4 p.205], with minor changes. Next, we build the Matrix function of the Green (G^ix,) operator on the segment , which is the solution of the boundary problem$\xi L^{(P,Q)}[x_{i-1}, x_{i+1}]$

$$\begin{cases} (P(x)\, G_x^{i}{}'(x,\xi))'_x - Q(x)\, \mathrm{G}^i(x,\xi) = 0\,,\ \xi \neq x\,, \\ \mathrm{G}^i(x_{i-1}) = \mathrm{G}^i(x_{i+1}) = 0\,,\ \mathrm{i} = \overline{-N+2, N-2}\,, \\ P(-1) G_x^{-N+1}(x,\xi)\,\big|_{x=-1} = P(1) G_x^{N-1}(x,\xi)\,\big|_{x=1} = 0\,. \end{cases} \quad \textbf{(1.8)}$$

And satisfying the conditions = 0 , = -$\left[\mathrm{G}^i(x,\xi)\right]_{x=\xi}\left[P(x)\, G_x^{i}{}'(x,\xi)\right]_{x=\xi}$E , where

$\left[\mathrm{G}^i(x,\xi)\right]_{x=\xi}$= .$\mathrm{G}^i(\xi + 0\,,\xi) - \mathrm{G}^i(\xi - 0\,,\xi)$

Lemma 3. Let the conditions (1,3) , (1,4) be satisfied , then the Green matrix function of the operator $\mathrm{G}^i(x,\xi)$L(P,Q) exists and has the form

$$\mathrm{G}^i(x,\xi)= \quad , \quad \begin{cases} V_1^i(\mathrm{x})\ \ [V_2^i(\mathrm{x}_{i-1})]^{-1} V_1^i(\xi)\ \ , x \le \xi \\ V_1^t(\mathrm{x})\ [V_2^i(\mathrm{x}_{i-1})]^{-1} V_1^i(\xi)\ \ , x \le \xi \\ \qquad x,\xi \epsilon [x_{i-1}, x_{i+1}] \end{cases} \quad \textbf{(1.9)}$$

Proof. The lemma is proved as in a uniformmesh tea (see, [3]).

2. **Accurate diagram.**

Definition. The exact three-point difference scheme (TTRS) for the problem (1.1), (1.2) let's call the difference scheme of the form

$\vec{y}_i$=++ ,$A_i[P(.), Q(.)]\vec{y}_{i+1} B_i[P(.), Q(.)]\vec{y}_{i-1}\vec{F}_i\left[P(.), Q(.), \vec{f}(.)\right]$

$B_{-N+1}\vec{y}_{-N}$== , i=-N+1 , N-1 . $A_{N-1}\vec{y}\vec{O}$ **(2.1)**

where the elements of the matrix coefficients , and the vector are functionals depending on $A_i B_i \vec{F}_i$ P(x), Q(x), ($\vec{f}$x) when x changes on the segment

$[x_{i-1}\,, x_{i+1}]$, and the conditions = , = and , in addition, the operator is linear on the third argument. $\vec{y}_i\, \vec{u}(x_i) i\overline{-N+1, N-1}\vec{F}_i\left[P(.), Q(.), \vec{f}(.)\right]$

Theorem 1 . Let the conditions (1.3) , (1.4) be met, then the TTPC for problem (1.1), (1.2) exists and can be represented as =- , =-.$(A\vec{u}_{\bar{x}})_{x,i} - D_i\vec{u}_i + \frac{1}{h_i}(A_i - B_i)\vec{u}_{\bar{x},i}\vec{F}_i i\overline{N+1\,, N-1}$

$B_{-N+1}\vec{u}_{\bar{x},N+1}$== . $\vec{O}$,$A_N\vec{u}_{\bar{x},N-1}O$**(2.2)**

где h_i =- ,) , ,$x_i x_{i-1} h_i = 0.5(h_i + h_{i+1} A_{i+1} = \left[h_{i+1}^{-1}\bar{V}_2^i(x_i)\right]^{-1}$

$B_i = \left[h_{i+1}^{-1}\bar{V}_1^i(x_i)\right]^{-1}$, $D_i = \hat{T}^i(Q)$, $\vec{F}_i$=$\hat{T}^i(\vec{f})$,

$$\hat{T}^i(W) = \frac{1}{h_i}\left\{[V_1^{-i}(x)]^{-1}\int_{x_{i-1}}^{x_i}\bar{V}_1^i(\eta)\,\mathrm{W}(\eta)d\eta + [V_2^{-i}(x)]^{-1} * \int_{x_i}^{x_{i+1}}\bar{V}_2^i(\eta)\,\mathrm{W}(\eta)d\eta\right\}$$

Proof. Multiply equation (1.1) on the left by Green's matrix function

$\mathrm{G}^i(x,\xi)$ and integrating the po from to after simple transformations, we obtain a difference scheme of the form (2.2), which is exact by construction. $\xi x_{i-1} x_{i+1}$

Lemma 4. Let the conditions (1.3) be met then when

V_j^i(x)=($\bar{V}_j^{i*}$x) , j=1.2; =-. $i\overline{N+1\,, N-1}$ **(2.3)**

ТТПС (1.13) is reduced to a divergent form

$(A\bar{u}_{\bar{x}})_{\hat{x}}$ -Dii$\vec{u}$ =-$\vec{F}$i , x , $\epsilon\widehat{\omega}_{h_i}$

$A_{-N+1}\vec{u}_{\bar{x},N+1}$==$A_{N-1}\vec{u}_{\bar{x},N-1}\vec{O}$

The proof is obvious.

3. **Truncated difference schemes. The accuracy of truncated circuits.**

Finding matrix solutions (V_j^ix) explicitly in the general case is not possible, so on the segment we enter the local coordinate system: $[x_{i-1}, x_{i+1}]$ x = , x$_i$ + sh*s = (x-)/x$_i h^*$

Where is $h^* = \begin{cases} h_i & \text{при } x \in [x_{i-1}, x_i], \\ h_{i+1} & \text{при } x \in [x_i, x_{i+1}]. \end{cases}$

Then the original segment is transformed into a segment [-1;1], the point s = 0 will correspond to the point x = .x_i

Put

$\bar{V}_j^i$(x)=()= $V_j^i x_i + sh_i \begin{cases} h_i\, \alpha^i(s, h_i) & , i = -\overline{N+2, N-1}, \\ \alpha^{-N+1}(s, h_{-N+1}), i = -N+1. \end{cases}$

$\bar{V}_2^i$(x)=()= $V_2^i x_i + sh_i \begin{cases} h_{i+1}\, \beta^i(s, h_i) & , i = -\overline{N+1, N-2}, \\ \beta^i(s, h_{N-1}), i = N-1. \end{cases}$

Шаблонные матричные функции $\alpha^i(s,h)$, are solutions to the corresponding Cauchy problems for and $\beta^i(s,h) V_1^i V_2^i$. We will look for these solutions in the form.

$\alpha^i(s,h)$= , (s,)=$\sum_{k=0}^{\infty} {h_i}^{2k} \alpha_k^i(s)$ $\beta^i h_{i+1} \sum_{k=0}^{\infty} {h_{i+1}}^{2k} \beta_k^i(s)$ **(3.1)**

где α_k^i, β_k^i determined by recurrence formulas

$\alpha_0^i(s)$= , ; ,$\int_{-1}^{s} \tilde{P}^{-1}(\xi)\, d\xi$i $= -\overline{N+2, N-1}\, \alpha_0^{-N+1} \equiv E$

$\alpha_k^i(s) = \int_{-1}^{s}\int_{-1}^{\xi} \tilde{Q}(t) \alpha_{k-1}^i(t) \tilde{P}^{-1}(\xi) dt\, d\xi$, , $i = -\overline{N+1, N-1}$ **(3.2)**

$\beta_0^i(s)$= , ; , $\int_{s}^{1} \tilde{P}^{-1}(\xi)\, d\xi$i $= -\overline{N+1, N-2}\, \beta_0^{N-1} \equiv E$

$\beta_k^i(s)$= , ; k=1,2,3,... $\int_{-1}^{s}\int_{-1}^{\xi} \tilde{Q}(t) \beta_{k-1}^i(t) \tilde{P}^{-1}(\xi) dt\, d\xi$i $= -\overline{N+1, N-1}$

If in the formulas (3.1) instead of , ($\alpha^i(s,h)\beta^i$s, put h_{i+1})

$\alpha^m(s, h_i)$= , = ,$\sum_{k=0}^{m} {h_i}^{2k} \alpha_k^i(s) \beta^m(s, h_i) \sum_{k=0}^{m} {h_{i+1}}^{2k} \beta_k^i(s)$

то instead of the exact diagram (2.2) we get a difference scheme of the form

()$A^m \overrightarrow{y_{\hat{x}}}$x,i-ii$D^m \vec{y}$+($\frac{1}{h_i} A^m$i-i$B^m$)=-$\overrightarrow{y_{\bar{x},i}} \vec{F}^m$i,$i = \overline{-N+1, N-1}$

$B_{-N+1}^m \vec{y}_{\bar{x},-N+1}$== , = $A_N^m \vec{y}_{\bar{x},N-1} \vec{O}\, i\overline{-N+1, N-1}$. **(3.3)**

гдеA_{i+1}^m = , $[\beta^m(0, h_{i+1})]^{-1} B^m$i= ,$[\alpha^m(0, h_i)]^{-1}$

$D^m{}_i$=($T^m \tilde{Q}$(s)) , =T($\vec{F}_i \vec{\tilde{f}}$(s)) ,

T^m((s)) = + ,$\widetilde{W}[\alpha^m(0, h_i)] \int_{-1}^{s} \alpha^m(\xi, h_i) \widetilde{W}(\xi)\, d\xi [\beta^m(0, h_{i+1})]^{-1} \int_{s}^{1} \beta^m(\xi, h_{i+1}) \widetilde{W}(\xi)\, d\xi$

$\widetilde{W}$(s)= W() . $x_i + sh^*$ **(3.4)**

The three-point difference scheme (3.3), (3.4) will later be called the truncated difference scheme of the m-th rank for the problem (1.1), (1.2) in the case of an uneven grid.

Let's study the properties of matrix functions , , . For the sake of brevity, here are only proofs of the properties of . , have similar properties and we will not give them separately. $\alpha^i(s, h_i), \beta^i(s, h_{i+1}) \alpha_k^i(s) \beta_k^i(s) \alpha^i(s, h_i), \alpha_k^i(s)$Матричные функции $\beta^i(s, h_{i+1}) \beta_k^i(s)$

Lemma 5. Suppose the conditions (1.3) ,(1.4) are satisfied, then >0 that at 0< () there are inequalities $\exists\, h_0 h_* \le h_0 h_* = \max h_i$

$\|\alpha^i(s, h_i)\| \le C(1 - x_i)^{-1}(1 + x_{i-1})^{-1}$, **(3.5)**

$\|\alpha^m(s, h_i)\| \le C(1 - x_i)^{-1}(1 + x_{i-1})^{-1}$, **(3.6)**

$\left\|[\alpha^i(s, h_i)]^{-1}\right\| \le C(1 - x_i)^{-1}(1 + x_{i-1})^{-1}$, **(3.7)**

$\|[\alpha^m(s, h_i)]^{-1}\| \le C(1 - x_i)^{-1}(1 + x_{i-1})^{-1}$, **(3.8)**

Proof. Prove inequality (3.5)

$\|\alpha^i(s, h_i)\| \le \sum_{k=0}^{\infty} {h_i}^{2k} \|\alpha^i(s)\|$ and $(\alpha_0^i(s)\vec{y}, \vec{y}) = \int_{-1}^{s} \frac{(\tilde{P}_1^{-1}(\eta)\vec{y}, \vec{y}) d\eta}{1 - (x_i + \eta h_i)^2} \le M\|\vec{y}\|^2 \int_{-1}^{s} \frac{d\eta}{1 - (x_i + \eta h_i)^2} \le \frac{C(s+1)}{(1 - x_i)(1 + x_{i-1})}$, .i $= -\overline{N+2, N-1}$

Следовательно , .$\|\alpha_0^i(s)\| \le M(1 - x_i)^{-1}(1 + x_{i-1})^{-1}(1 + s)$

$\left\|\alpha_0^i(s)\right\|$ =* where , , that follows from the conditions (1.4) $\left\|\int_{-1}^{s}\int_{-1}^{\eta}\tilde{Q}(t)\alpha_0^i(s)\tilde{P}_1^{-1}(\eta)dtd\eta\right\| \leq \int_{-1}^{s}\frac{\|\tilde{P}_1^{-1}(\eta)\|}{1-(x_i+\eta \mathrm{h}_i)^2}\int_{-1}^{\eta}\tilde{Q}(t)\left\|\alpha_0^i(s)\right\|dtd\eta \leq M^2\cdot M(1-x_i)^{-2}(1+x_{i-1})^{-2}\ \frac{(1+\mathrm{s})}{3!}\left\|\tilde{P}_1^{-1}(x)\right\| \leq M\left\|\tilde{Q}(x)\right\| \leq M_1$

Далее applying mathematical induction, we find that

$\left\|\alpha_k^i(s)\right\| \ \leq M^{k+1}M_1^k(1-x_i)^{-k-1}(1+x_{i-1})^{-k-1}\frac{(1+s)^{2k+1}}{(2k+1)!}$, i= ; k=1,2,3...$\overline{2\ ,N-1}$

Using the obtained inequalities, we estimate the norm $\alpha^i(s,\mathrm{h}_i)$

$\left\|\alpha^i(s,\mathrm{h}_i)\right\| \leq \sum_{k=0}^{\infty}h_i^{\,2k}\ \left\|\alpha_k^i(s)\right\| \leq$ M $(1-x_i)^{-1}(1+x_{i-1})^{-1}\left[1+s+\frac{h_i^{\,2}M\cdot M_1\cdot(1+s)^3}{(1-x_i)(1+x_{i-1})\cdot 3!}+\left(\frac{h_i^{\,2}M\cdot M_1}{(1-x_i)(1+x_{i-1})}\right)^2\frac{(1+s)^5}{5!}+\cdots+\left(\frac{h_i^{\,2}M\cdot M_1}{(1-x_i)(1+x_{i-1})}\right)^k\frac{(1+s)^{2k+1}}{(2k+1)!}+\cdots\right]$.

Так как $\frac{h_i^{\,2}}{(1-x_i)(1+x_{i-1})} \leq \frac{h_i}{2(1-h_i)}$,

при 0 meets the conditions <1 and the series in square brackets is majorized next to $\leq h_i \leq 2(M\cdot M_1+2)h_i^{\,2}\cdot\ M\cdot M_1/((1-x_i)*(1+x_{i-1}))$

1+s+ +...+$\frac{(1+s)^3}{3!}\frac{(1+s)^{2k+1}}{(2k+1)!}+\cdots$

суммой которого является функции f(x)=0.5(exp(1+s)-exp(-1-s)).

It is not difficult to verify that this function will be incremental for any s∈[−1,0] and $\max\limits_{s\in[-1,0]} f(x)$= f (0)=0.5(e-)=$e^{-1}k_1$

Следовательно , . $\left\|\alpha^i(s,\mathrm{h}_i)\right\| \leq \mathrm{C}(1-\mathrm{x_i})^{-1}(1+x_{i-1})^{-1}$Since,

$\|\alpha^m(s,\mathrm{h}_i)\| \leq \left\|\alpha^i(s,\mathrm{h}_i)\right\|$ inequality follows from proven inequality (3.6).

Оценка сверху для нормы матрицы $\left\|\alpha^i(0,\mathrm{h}_i)\right\|^{-1}$ it follows from the fact that, taking into account the estimates for the norm, it is possible to write down $\alpha_k^i(0,\mathrm{h}_i)\left\|\left[\alpha^i(0,\mathrm{h}_i)\right]^{-1}\right\| \leq \left\|\left[\alpha_k^i(0,\mathrm{h}_i)\right]^{-1}\right\|\ \|[E+o(\mathrm{h}_i)]^{-1}\|$

где $\left[\alpha_o^i(0,\mathrm{h}_i)\right]^{-1}=\left[\tilde{P}_1^{-1}(s)\int_{-1}^{s}\frac{d\eta}{1-(x_i+\eta\mathrm{h}_i)^2}\ +\int_{-1}^{s}\frac{[\tilde{P}_1^{-1}(\eta)-\tilde{P}_1^{-1}(s)]d\eta}{1-(x_i+\eta\mathrm{h}_i)^2}\right]^{-1} = \left[\int_{-1}^{s}\frac{d\eta}{1-(x_i+\eta\mathrm{h}_i)^2}+\widetilde{P}_1^{-1}(s)\int_{-1}^{s}\frac{[\tilde{P}_1^{-1}(\eta)-\tilde{P}_1^{-1}(s)]d\eta}{1-(x_i+\eta\mathrm{h}_i)^2}\right]^{-1}\tilde{P}_1^{-1}(s)$.

Переходя to the norms in the resulting equality at s=0 we have

$$\left\|\left[\alpha_o^i(s,\mathrm{h}_i)\right]^{-1}\right\| \leq M\left[\int_{-1}^{s}\frac{d\eta}{1-(x_i+\eta\mathrm{h}_i)^2}\right]^{-1}\ \|[E+A]^{-1}\|$$

где A =A()=(0)0, $\mathrm{h}_i\ \tilde{P}_1\ \int_{-1}^{0}\left[\frac{[\tilde{P}_1^{-1}(\eta)-\tilde{P}_1^{-1}(s)]d\eta}{1-(x_i+\eta\mathrm{h}_i)^2}\right]^{-1}$ и

$$\frac{s+1}{(1-x_{i-1})(1+x_i)} \leq \int_{-1}^{s}\frac{d\eta}{1-(x_i+\eta\mathrm{h}_i)^2} \leq \frac{s+1}{(1-x_i)(1+x_{i-1})}$$

It is known that if <1 , then = = $\|A\|[E+A]^{-1}\ [E-A+A^2-A^3+A^4-\cdots]^{-1}\{[-A+A^2-\ A^3+A^4-\cdots][E+A]\}^{-1}\mathrm{E}$.

Следовательно , ,$\|[E+A]^{-1}\| \leq 1+\|A\|+\|A\|+\cdots=\frac{1}{1-\|A\|}$

what can be achieved if you require the continuity of the matrix <1 that when <, the inequality is fulfilled $\tilde{P}_1^{-1}(s)$, тогда ∃ такое $0<\mathrm{h}_0 0<\mathrm{h}_*\mathrm{h}_0$

$\left\|\tilde{P}_1^{-1}(\eta)-\tilde{P}^{-1}(s)\right\| \leq o(1)$ for $\forall\,\eta,s\in[-1,0]$, с учетом выше сказанного имеем

$\left\|\left[\alpha_o^i(s,\mathrm{h}_i)\right]^{-1}\right\| \leq \left\|\widetilde{P}_1(s)\right\|(1-x_{i-1})(1+x_i)\left[1-\left\|\widetilde{P}_1(s)\right\| * \max\limits_{s\in[-1,0]}\left\|\tilde{P}_1^{-1}(\eta)-\tilde{P}_1^{-1}(s)\right\|\right]$, or

$\left\|\left[\alpha_o^i(s,\mathrm{h}_i)\right]^{-1}\right\| = \left\|\left\{E + \left[\alpha_o^i(s,\mathrm{h}_i)\right]^{-1}\left[\sum_{j=1}^{\infty} h_i{}^{2j}\left[\alpha_j^i(0,\mathrm{h}_i)\right]^{-1}\right]\right\}\right\| \leq \left[1 - \right.$
$\left\|\alpha_o^i(s,\mathrm{h}_i)\right]^{-1}\left\|\sum_{j=1}^{\infty} h_i{}^{2j}\left[\alpha_j^i(0,\mathrm{h}_i)\right]^{-1}\right\|\left[\alpha_o^i(s,\mathrm{h}_i)\right]^{-1}\right\| \leq \mathrm{M}(1-x_{i-1})(1+x_i)$.

Аналогично доказывается неравенство (3.8) . В дальнейщем потребуем , непрерывность по Гельдеру со степенью μ $(0 < \mu \leq 1)$ matrices $\tilde{P}_1^{-1}(x)$, and Q(x) i.e. inequalities are being met

$\left\|\tilde{P}_1^{-1}(x) - \tilde{P}_1^{-1}(y)\right\| \leq K_1|x-y|^{\mu}$, $\|Q(x) - Q(y)\| \leq K_2|x-y|^{\mu}$ for $\forall\, x, y \in (-1,1)$,
K_1, K_2=const . **(3.9)**

Atμ =1 , matrices are called Lipschitz continuous.

Let's introduce the following scalar works on the grid $\widehat{\omega}_{h_i}$

$(\vec{u}\,,\vec{v})_{h_i} = .\sum_{i=-N+1}^{N-1}(\vec{u}_i\,,\vec{v}_i)\,h_i$

где under the sign of the sum is the usual scalar product of vectors in R^n .

Если enter notation

$\Omega_{i,1}{}^{(m+1)}(s,) = =-,\, h_i\sum_{k=0}^{\infty} h_i{}^{2(k+m+1)}\alpha_{k+m+1}^i\,(s)\ \alpha^i(s,\mathrm{h}_i)\alpha^{(m)}(s,\mathrm{h}_i)$

$\Omega_{i,2}{}^{(m+1)}(s,) = \ h_{i+1}\sum_{k=0}^{\infty} h_{i+1}{}^{2(k+m+1)}\beta_{k+m+1}^i\,(s) = \beta^i(s,\mathrm{h}_{i+1}) - \beta^{(m)}(s,\mathrm{h}_i)$.

then for matrices

$\Omega_{i,j}{}^{(m+1)}(.,.)$ (j=1,2,) $\mathrm{i} = -\overline{N+1\,,N-1}$ имеют место следующие неравенства

$$\left\|\Omega_{i,1}^{(m+1)}(\mathrm{s},h_i)\right\| \leq \begin{cases} C(1-x_i)^{-1}(1+x_{i-1})^{-1}N^{-(2m+2)}\ ,\mathrm{i} = -\overline{N+2\,,N-1} \\ CN^{-(2m+2)}\,, \qquad i = -N+1 \end{cases},$$

$$\left\|\Omega_{i,2}^{(m+1)}(\mathrm{s},h_{i+1})\right\| \leq \begin{cases} C(1+x_i)^{-1}(1-x_{i+1})^{-1}N^{-(2m+2)}\ ,\mathrm{i} = -\overline{N+1\,,N-2} \\ CN^{-(2m+2)}\,, \qquad i = N-1 \end{cases},$$

These inequalities are proved as analogous inequalities from the work [5] taking into account the unevenness of the grid.

The following occurs:

Lemma 6. Suppose the conditions (1.3) , (1.4) are satisfied. Then there is such a h_0>0 that , at 0< () $h_* \leq h_0 h_* = \max\{h_i\}$the inequalities are satisfied

$(D\,\vec{v},\vec{v})_{h_i} + (A\,\vec{v}_{\bar{x}},\vec{v}_{\bar{x}})_{h_i} \geq C\left(\left\|(1-x^2)^{\frac{1}{2}}\vec{v}_{\bar{x}}\right\|_{h_i} + \|\vec{v}\|_{h_i}\right)$ **(3.10)**

$\left|\left(\left(A^{(m)} - B^{(m)}\right)\vec{v}_{\bar{x}},\vec{v}\right)_{h_i}\right| \leq Ch_*\left(\left\|(1-x^2)^{\frac{1}{2}}\vec{v}_{\bar{x}}\right\|_{h_i} + \|\vec{v}\|_{h_i}\right)$ **(3.11)**

$\left\|A_i - A_i^{(m)}\right\| \leq\ C(1-x^2{}_i)N^{-(2m+2)}$ **(3.12)**

$\left\|B_i - B_i^{(m)}\right\| \leq\ C(1-x^2{}_i)N^{-(2m+2)}$ **(3.13)**

$\left\|D_i - D_i^{(m)}\right\| \leq CN^{-(2m+2)}$, $\left\|\vec{F}_i - \vec{F}_i^{(m)}\right\| \leq CN^{-(2m+2)}$ **(3.14)**

If you additionally require the continuity of The Gelder matrices $P^{-1}(x)$ and Q(x) with a degree $\mu(0 < \mu \leq 1)$ then ,

$\left|\left(\left(A^{(m)} - B^{(m)}\right)\vec{v}_{\bar{x}},\vec{v}\right)_{h_i}\right| \leq Ch_*{}^{1+\mu}\left(\left\|(1-x^2)^{\frac{1}{2}}\vec{v}_{\bar{x}}\right\|_{h_i} + \|\vec{v}\|_{h_i}\right)$ **(3.15)**

Proof. Inequalities(3.10)-(3.15) are proved as similar inequalities from the work [5] taking into account the unevenness of the grid.

Using lemmas 5 and 6, we will prove the following basic theorem on the convergence rate of truncated difference schemes of the m-th rank (3.3).

Theorem 2. Suppose the conditions (1.3) and (1.4) are satisfied and the matrices $\mathrm{P}^{-1}(x)$, Q(x) are continuous according to Gelder with a degree $\mu(0 < \mu \leq 1)$. Then there exists such (h_0) $h_0 > 0$that at 0<$h_* \leq h_0$ truncated difference scheme of the m-th rank (3.3) has an accuracy of 0() i.e.$N^{-(2m+2\mu)}$
fair inequalities

$\|\vec{u}-\vec{y}\|_{V_{h_i}} \leq C\,N^{-(2m+2\mu)})(1+)$ $\|\vec{y}\|_{V_{h_i}}$ **(3.16)**

where V_{h_i} is the differenceanalogy of space V in the case of an uneven grid .

$(\vec{u},\vec{v})_{V_{h_i}}$=+$\left[(1-x^2)^{\frac{1}{2}}\vec{u}_{\bar{x}}\,,\vec{v}_{\bar{x}}\,\right]_{h_i}(\vec{u},\vec{v})_{h_i}$

$\|\vec{v}\|_{V_{h_i}}=\left((\vec{u},\vec{v})_{V_{h_i}}\right)^{1/2}$

Proof. Let $\vec{u}$ (x) be the solution of the exact scheme of the boundary problem (1.1), (1.2), $\vec{y}$(x) be the solution of the corresponding truncated difference scheme (3.3). Through $\vec{z}=\vec{u}-\vec{y}$ the denotation of the error of the truncated difference scheme (3.3). Assuming $\vec{u}=\vec{z}+\vec{y}$ in the exact scheme (2.2) we getthe following problem for the error $\vec{z}$.

$(A\,\vec{z}_{\bar{x}})_{\hat{x},i}$ -+ =+ $D_i\vec{z}_i\frac{1}{h_i}(A_i-B_i)\vec{z}_{\hat{x},i}\left(\left(A_i^{(m)}-A_i\right)\vec{y}_{\bar{x}}\right)_{\hat{x},i}\frac{1}{h_i}\left(A_i^{(m)}-A_i+B_i-B_i^{(m)}\right)\vec{y}_{\hat{x},i}$ +- $\left(D_i-D_i^{(m)}\right)\vec{y}_i+$ $\vec{F}_i^{(m)}\vec{F}_i$, ,i $=-\overline{N+1\,,N-1}$

$B_{-N+1}\vec{z}_{\hat{x},-N+1}=\vec{0}$, .$A_N\vec{z}_{x,N-1}=\vec{0}$

Scalarly $\vec{z}$ multiplying the resulting expression by and applying the formula of summation in parts , the Cauchy-Boone yakovsky inequality, using the method of energy inequalities [4], after simple transformations we obtain a proof of the theorem.

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