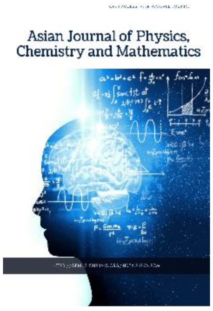

A Review on the Integral Transforms

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ABSTRACT

Many integral transforms have been proposed, tested, and proven efficient in solving many applications in various scientific fields. The diversity of integral transforms comes from their outstanding ability to solve problems by converting them from one domain where the solution is done through a complex mathematical procedure to another domain where simple algebraic methods can solve them. This work demonstrates the chronological diversity of integral transforms by displaying a collection of integral transforms with their basic properties.

Keywords: Integral transform, Laplace, Sumudu, Natural, Al-Tememe, Elzaki, Aboodh, Kashuri and Fundo, ZZ, Ramadan Group, Mahgoub, Kamal, Polynomial, Mohand, Rangaig, Al-Zughair, Sadiq, Extension of Al-Zughair, Rohit, Dinesh Verma, Gupta, SEE, Alenezi, Emad-Sara, Complex SEE, Natural Logarithmic, Soham.

1. Introduction

The mathematical technique responsible for transforming differential equation into algebraic equation is referred to as the integral transform, and this procedure is responsible for converting a complicated problem into a simpler one to be solved mathematically. The integral transform as a mathematical operator can produce a result function by integrating the product of a function with another function called the kernel of the integral transform. The general format of the integral transform can be written as follows:

$$F(s) = \int k(x,s) f(x) dx.$$

Where, $F(s)$ is the function resulting from the integral transform and $k(x,s)$ is the kernel function.

Integral transform can convert function from one domain where some mathematical procedures are fairly difficult into another domain where they become more malleable and mathematically smoother to process. The resulting function is usually transformed back into its original domain via an inverse form of the used integral transform [1,2].

In 1763, Euler introduced the integral transformation to the world for the first time [3]. Since that time, mathematicians have been eager to invest their time and effort to investigate, propose, and test the capabilities of using existing and new integral transforms in different aspects of life applications [4-7].

Over the years, many integral transforms have been proposed, and most of these transforms have been named after the mathematicians who proposed them. Some integral transforms and their properties have been chronologically displayed in this work.

2. Integral Transforms

This section displays integral transforms and their properties arranged based on the chronology of their appearance from the oldest to the most recent.

2.1. Laplace Transformation [8]

This transform had been developed by Laplace in 1780, and it is considered one of the oldest and most popular integral transforms.

Laplace transform is defined for the function $f(t)$ as:

$\mathcal{L}\{f(t)\} = \int_{t=0}^{\infty} f(t)e^{-st}dt = F(s)\, , R_e(s) > 0.$

The basic properties of Laplace transformation are:

1) $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}, n \geqslant 0.$
2) $\mathcal{L}\{e^{at}\} = \frac{1}{s-a}$, a is a constant.
3) $\mathcal{L}\{\sin(at)\} = \frac{a}{s^2+a^2}.$
4) $\mathcal{L}\{\cos(at)\} = \frac{s}{s^2+a^2}.$
5) $\mathcal{L}\{\cosh(at)\} = \frac{s}{s^2-a^2}.$
6) $\mathcal{L}\{\sinh(at)\} = \frac{a}{s^2-a^2}.$
7) $\mathcal{L}\{f^{(n)}(t)\} = s^nF(s) - \sum_{k=0}^{n-1} s^{n-k-1}f(0).$

2.2. Sumudu Transformation [9]

G. K. Watugala introduced this transform in 1993. The integral transform is defined for a function $f(t)$ as:

$S_u\{f(t)\} = \int_{t=0}^{\infty} f(vt)e^{-t}dt = F(N), v \in (-k_1, k_2)$ and $k_1, k_2 > 0.$

The Basic properties of Sumudu transformation are:

1) $S_u\{t^n\} = n!\, v^n\, , n \geqslant 0.$
2) $S_u\{e^{at}\} = \frac{1}{1-av}$, a is a constant.
3) $S_u\{\sin(at)\} = \frac{av}{1+a^2v^2}.$
4) $S_u\{\cos(at)\} = \frac{1}{1+a^2v^2}.$
5) $S_u\{\sinh(at)\} = \frac{av}{1-a^2v^2}.$
6) $S_u\{\cosh(at)\} = \frac{1}{1-a^2v^2}.$
7) $S_n\{f^{(n)}(t)\} = \frac{F(v)}{v^n} - \sum_{n-k}^{n-1} \frac{f(0)}{v^{n-k}}.$

2.3. Natural Transformation [10]

In 2008, Khan, Z.H. et al. introduced an integral transform named the Natural transform. The integral transform is defined for the function $f(t)$ as:

$N^+\{f(t)\} = \int_{t=0}^{\infty} f(vt)e^{-st} = F(v,s)\, ,\ v, s \in (-k_1, k_2)$ and $k_1, k_2 > 0.$

The basic properties of Natural transformation are:

1) $N^+\{t^n\} = \frac{n!v^n}{s^{n+1}}, n \geqslant 0.$
2) $N^+\{\mathrm{e}^{\mathrm{at}}\} = \frac{1}{s+av}$, a is a constant.
3) $N^+\{\sin(at)\} = \frac{av}{s^2+a^2v^2}.$
4) $N^+\{f^{(n)}(t)\} = \frac{s^n}{v^n}F(v,s) - \sum_{k=0}^{n-1} \frac{s^{n-k-1}}{v^{n-k}} f^{(k)}(0).$

2.4. Al-Tememe Transformation [11]

Another transform introduced in 2008 by Ali Hassan Mohammed et al. is called the Al-Tememe transform. Al-Tememe transform is convergent, and it is defined for a function $f(t)$ as:

$T\{f(t)\} = \int_{t=1}^{\infty} t^{-p}f(t)dt = F(p)\ , (t > 1)$ and p is a parameter.

The basic properties of Al-Tememe transform are:

1) $T(k) = \frac{k}{p-1}, p > 1$, and k is a constant.
2) $T\{\ln(d)\} = \frac{1}{(p-1)^2}, p > 1.$
3) $T\{t^n\ln(t)\} = \frac{1}{(p-(n+1))^2}, p > (n+1)\, ,$ $n \in \mathbb{R}.$
4) $T\{sin(a\, ln(t))\} = \frac{a}{(p-1)^2+a^2}, p > 1$, a is a constant.
5) $T\{\cos(a\, ln(t))\} = \frac{(p-1)}{(p-1)^2+a^2}, p > 1$, and a is a constant.
6) $T\{t^nf^{(n)}(t)\} = -f^{(n-1)}(1) - (p - n)f^{(n-2)}(1) - \cdots - (p-n)(p-(n-1)) \ldots - (p-2)f(1) + (p-n)!\, F(p).$

2.5. Elzaki Transformation [12]

In 2011, Tarig. M. Elzaki introduced Elzaki integral transform. The integral transform is defined for a function $f(t)$ as:

$E\{f(t)\} = v\int_{t=0}^{\infty} f(t)e^{-\frac{t}{v}}dt = f(v), v \in (k_1, k_2)$, $k_1, k_2 > 0$.

The basic properties of Elzaki transformation are:

1) $E\{t^n\} = n!\, v^{n+2}$, $n \geqslant 0$.
2) $E\{e^{at}\} = \frac{v^2}{1-av}$, a is a constant.
3) $E\{\sin(at)\} = \frac{av^3}{1+a^2v^2}$.
4) $E\{\cos(at)\} = \frac{v^2}{1+a^2v^2}$.
5) $E\{\sinh(at)\} = \frac{av^3}{1-a^2v^2}$.
6) $E\{\cosh(at)\} = \frac{v^2}{1-a^2v^2}$.
7) $E\{f^{(n)}(t)\} = \frac{F(v)}{v^n} - \sum_{k=0}^{n-1} v^{2mn+k} f^{(k)}(0)$.

2.6. Aboodh Transformation [13]

Khalid Suliman Aboodh introduced in 2013 an integral transform called the Aboodh integral transform. The integral transform is defined for a function $f(t)$ as: $A\{f(t)\} = F(v) = \frac{1}{v}\int_{t=0}^{\infty} f(t)e^{-vt}dt$, $t \geqslant 0$, $k \leqslant v \leqslant k_2$, $k_1, k_2 > 0$.

The basic properties of Aboodh transformation are:

1) $A\{t^n\} = \frac{n!}{v^{n+2}}$, $n = 0,1,2,3,\ldots$
2) $A\{e^{\mathrm{at}}\} = \frac{1}{v^2-av}$, a is a constant.
3) $A[\sin(at)\} = \frac{a}{v(v^2+a^2)}$.
4) $A\{\cos(a+1)\} = \frac{1}{v^2+a^2}$
5) $A\{\sin k(at)\} = \frac{a}{v(v^2-a^2)}$.
6) $A(\cosh(at)) = \frac{v(v^2-a^2)}{v^2-a^2}$.
7) $A\{f^{(n)}(t)\} = v^n F(v) - \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{v^{2-n+k}}$.

2.7. The Integral Transformation by Kashuri and Fundo [14]

In 2013, Kashuri A. et al. introduced an integral transform. The integral transform is defined for a function $f(t)$ as:

$K\{f(t)\} = \frac{1}{v}\int_{t=0}^{\infty} f(t)e^{-\frac{t}{v^2}}dt = F(v)$, $v \in (-k_1, k_2)$ and $k_1, k_2 > 0$.

The basic properties of the integral transformation are:

1) $K\{t^n\} = n!\, v^{2m+1}, n \geqslant 0$.
2) $K\{e^{at}\} = \frac{V}{1-av^2}$, a is a content.
3) $K\{\sin(a)\} = \frac{av^2}{1+a^2v^4}$.
4) $K\{\cos(at)\} = \frac{v}{1+a^2v^4}$.
5) $K\{\cosh(at)\} = \frac{v}{1-a^2v^4}$.
6) $K\{\sinh(a+1)\} = \frac{av^2}{1-a^2v^4}$.
7) $K\{f^{(n)}(t)\} = \frac{1}{v^{2n}}F(v) - \sum_{k=0}^{n-1} \frac{f^{(k)}(0)}{v^{2(n-k)-1}}$.

2.8. ZZ Integral Transform [15]

In 2016, Zain UlAbadin Zafar introduced an integral transform called the ZZ transform. For an exponential order function $f(t)$, the integral transform is defined as follows:

$H\{f(t)\} = \mathrm{s}\int_{t=0}^{\infty} f(vt)e^{-st}dt = F(s, v)$, $t \geqslant 0$.

The basic properties of ZZ transformation are:

1) $H\{t^n\} = n!\frac{v^n}{s^n}, n \geqslant 0$.
2) $H\{e^{\mathrm{at}}\} = \frac{s}{s-av}$, a is a constant.
3) $H\{\sin(a+1)\} = \frac{avs}{s^2+a^2v^2}$.
4) $H\{\cos(at)\} = \frac{v^2}{1+a^2v^2}$.
5) $H\{f^{(n)}(t)\} = \frac{s^n}{v^n}F(s,v) - \sum_{k=0}^{n-1} \frac{s^{n-k}}{v^{n-k}} f^{(k)}(0)$.

2.9. Ramadan Group (RG) Transform [16]

Mohamed A. Ramadan et al. proposed an integral transform in 2016 called the Ramadan Group (RG) transform. RG transformation is defined for a function $f(t)$ as:

$RG\{f(k)\} = \int_{t=0}^{\infty} f(vt)e^{-st}dt = F(v,s)$, $v \in (k_1, k_2)$ and $k_1, k_2 > 0$.

The basic properties of RG transformation are:

1) $RG\{t^n\} = \frac{n!}{v^{n+1}}, n \geqslant 0$.
2) $RG\{e^{\mathrm{at}}\} = \frac{1}{s+av}$, a is a constant.
3) $RG\{\sin(at)\} = \frac{av}{s^2+a^2v^2}$.
4) $RG\{f^{(n)}(t)\} = \frac{s^n}{v^n}F(v) - \sum_{k=0}^{n-1} \frac{s^{n-k-1}}{v^{n-k}} f^{(k)}(0)$.

2.10. Mahgoub Transform [17]

Another transform emerged in 2016 by Mohand M. et al. called Mahgoub transform. The integral transform is defined for a function $f(t)$ as:

$M\{f(t)\} = F(v) = v\int_{t=0}^{\infty} f(t)e^{-vt}dt, t \geqslant 0\ k_1, < v \leqslant k_2$ and $k_1, k_2 > 0$.

The basic properties of Mahgoub transformation are:

1) $M\{t^n\} = \frac{n!}{V^n}, n \geqslant 0$.
2) $M[e^{at}] = \frac{v}{v-a}$, a is a constant.
3) $M\{\sin(at)\} = \frac{av}{v^2+a^2}$.
4) $M\{\cos(at)\} = \frac{v^2}{v^2+a^2}$.
5) $M\{\sinh(at)\} = \frac{av}{v^2-a^2}$.
6) $M\{\cosh(at)\} = \frac{v^2}{v^2-a^2}$.
7) $M\{f^{(n)}(t)\} = v^nF(v) - \sum_{k=0}^{n-1} v^{n-k}f^{(k)}(0)$.

2.11. Kamal Transform [18]

In 2016, Abdelilah Kamal et al. presented the Kamal transform. The integral transform is defined for a function $f(t)$ as:

$k\{f(t)\} = F(v) = \int_{t=0}^{\infty} f(t)e^{-t}dt, t \geqslant 0, k \leqslant v \leqslant k_2$ and $k_1, k_2 > 0$.

The basic properties of Kamal transformation are:

1) $k\{t^n\} = n!\, v^{n+1}$, $n \geqslant 0$.
2) $k\{e^{at}\} = \frac{v}{1-av}$, a is a constant.
3) $k\{\sin(at)\} = \frac{av^2}{1+a^2v^2}$.
4) $k\{\cos(at)\} = \frac{v}{1+a^2v^2}$.
5) $k\{\sinh(at)\} = \frac{av^2}{1-a^2v^2}$.
6) $k\{\cosh(at)\} = \frac{v}{1-a^2v^2}$.
7) $k\{f^{(n)}(t)\} = v^{-n}F(v) - \sum_{k=0}^{n-1} v^{k-n+1}f^{(k)}(0)$.

2.12. Polynomial Transform [19]

2016 was such an eventful year for integral transforms. Benedict Barnes proposed the Polynomial integral transform. The integral transform is defined for a function $f(t)$ as:

$B\{f(t)\} = F(v) = \int_{t=1}^{\infty} f(\ln(t)) \cdot t^{-v-1}dt$, $t \in [1, \infty)$ and $t \geqslant 1$.

And $B\{f^{(n}(t)\} = V^nF(v) - V^{n-1}f(0) - V^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$.

where $F(v) = B\{f(t)\}$.

2.13. Mohand Transform [20]

In 2017, Mohand M. et al. proposed an integral transform called the Mohand transform. The integral transform is defined for a function $f(t)$ as:

$M_o\{f(t)\} = F(v) = v^2\int_{t=0}^{\infty} f(t)e^{-vt}dt, t \geqslant 0\ k_1, \leqslant v \leqslant k_2\ and\ \ k_1k_2 > 0$.

The basic properties of Mohand transformation are:

1) $M_o\{t^n\} = \frac{n!}{v^{n-1}}, n \geqslant 0$.
2) $M_o\{e^{at}\} = \frac{v^2}{v-a}$, a is a constant.
3) $M_o\{\sin(at)\} = \frac{av^2}{v^2+a^2}$.
4) $M_o\{\cos(at)\} = \frac{v^3}{v^2+a^2}$.
5) $M_o\{\sinh(at)\} = \frac{av^2}{v^2-a^2}$.
6) $M_o\{\cosh(a+1)\} = \frac{v^3}{v^2-a^2}$.
7) $M_o\{f^{(n)}(t)\} = v^nF(v) - \sum_{k=0}^{n-1} v^{n-k+1}f^{(k)}(0)$.

2.14. Rangaig Transform [21]

Rangaig integral transform is another transform emerged in 2017. The transform is defined for a function $f(t)$ as:

$\eta\{f(t)\} = F(v) = \int_{t=-\infty}^{0} e^{vt}f(t)dt$, $\frac{1}{\lambda_1} \leqslant v \leqslant \frac{1}{\lambda_2}\ and\ \lambda_1, \lambda_2 > 0$.

The Basic properties of Rangaig transformation are:

1) $\eta\{t^n\} = \frac{(-1)^n n!}{v^{n+2}}, n \geqslant 0$.
2) $\eta\{e^{at}\} = \frac{1}{v(v+a)}$.
3) $\eta\{\sin(t)\} = \frac{1}{v}\left(\frac{1}{v^2+1}\right)$.
4) $\eta\{\cos(t)\} = \frac{1}{v^2+1}$.
5) $\eta\{f^{(n)}(t)\} = (-1)^n v^n f(v) + (-1)^{n+1}\sum_{k=0}^{n-1}(-1)^k v^{n-2-k}f^{(k)}(0)$.

2.15. Al-Zughair Transform [22]

In 2018, the AL-Zughair integral transform was proposed by Ali H.M. et al. The integral transform is convergent and is defined as follows for a function $f(t)$:

$Z\{f(t)\} = \int_{t=1}^{e} \frac{[\ln(t)]^v}{t} f(t)dt = F(v)$.

Where v is a constant greater than (-1). The basic properties of AL-Zughair Transformation are:

1) $Z\{(\ln(t))^n\} = \frac{1}{v+(n+1)}$, $n \in \mathbb{R}$, $v > -(n+1)$.
2) $Z\{(\ln(\ln(t)))^n\} = \frac{(-1)^n n!}{(v+1)^{n+1}}, v > -1, n \in \mathbb{Z}^+$.
3) $Z\{\sin(a\, ln(ln(\theta))\} = \frac{-a}{(v+1)^2+a^2}, v > -1$, a is a constant.
4) $z\{\cos(a \ln(\ln(t)))\} = \frac{v+1}{(v+1)^2+a^2}, v > -1$, a is a constant.
5) $z\{\sinh(a \ln(\ln(t)))\} = \frac{-a}{(v+1)^2-a^2}, |v+1| > a$, a is a constant.
6) $z\{siash(a \ln(\ln(t)))\} = \frac{v+1}{(v+1)^2-a^2}, |v+1| > a$, a is a constant.
7) $\frac{d^k}{dv^k} Z\{f(t)\} = Z\{(\ln(\ln(t)))^{\mathrm{k}} \cdot f(t)\}, k = 1,2,3, \dots$

2.16. Sadiq Transform [23]

The Sadiq Integral transform is another transform proposed in 2018 by S.L. Shaikh. The integral transform is defined for a function $f(t)$ as:

$S_a\{f(t)\} = F(v^\alpha, \beta) = \frac{1}{v^\beta}\int_{t=0}^{\infty} e^{-tv^\alpha} f(t)dt.$

Where v is a complex variable, α is any nonzero real numbers, and β is any real number.

The basic properties of Sadiq transformation are:

1) $S_a\{t^n\} = \frac{n!}{V^{n\alpha+(\alpha+\beta)}}$, $n \geqslant 0$.
2) $S_a\{e^{\mathrm{at}}\} = \frac{v^{-\beta}}{v^\alpha - a}$, a is a constant.
3) $S_a\{\sin(at)\} = \frac{av^{-\beta}}{v^{2\alpha}+a^2}$.
4) $S_a\{\cos(at)\} = \frac{v^{\alpha-\beta}}{v^{2\alpha}+a^2}$.
5) $S_a\{\sinh(at)\} = \frac{av^{-\beta}}{v^{2\alpha}-a^2}$.
6) $S_a\{\cosh(at)\} = \frac{v^{\alpha-\beta}}{v^{2a}-a^2}$.
7) $S_a\{f^{(n)}(t)\} = v^{n\alpha}F(v) - \sum_{k=0}^{n-1} v^{k\alpha-\beta} f^{(n-1)-k}(0)$.

2.17. An Extension of AL-Zughair Transform [24]

In 2019, Ali H.M. et al. proposed an integral transform that is an extension of the AL-Zughair integral transform. For a function $f(|ln(t)|)$, the integral transform is defined as follows:

$EZ\{f(|\ln(t)|)\} = \int_{\mathrm{e}^{-1}}^{e} \frac{(\ln(t))^v}{t} f(|\ln(t)|)dt = F(V)$.

The basic properties of the extension of Al-Zughair transformation are:

1) $EZ\{(\ln(t))^n\} = \frac{2}{v+(n+1)}$, $(v+n)$ is an even number $n \geq 0$.
2) $EZ\{\ln(\ln(t))\} = \frac{-2}{(v+1)^2}$, v is an even number.
3) $EZ\{(\ln(\ln(t)))^n\} = \frac{(-1)^n 2*n!}{(v+1)^{n+1}}, v > -1, n = 1,2,3, \dots$.
4) $EZ\{\sin(a \ln(\ln(t))\} = \frac{-2a}{(v+1)^2+a^2}$, v is an even number, a is a constant, $v > -1, a \in \mathbb{R}$.
5) $EZ\{\cos(a \ln(\ln(t)))\} = \frac{2(v+1)}{(v+1)^2+a^2}$, v is an even number, $v > -1, a \in \mathbb{R}$.
6) $EZ\{\sinh(a \ln(\ln(t))\} = \frac{-2a}{(v+1)^2-a^2}$, v is an even number, $|v+1| > a, a \in \mathbb{R}$.
7) $EZ\{\cosh(a \ln(\ln(t)))\} = \frac{2(v+1)}{(1+1)^2-a^2}$, v is an even number, $|v+1| > a, a \in \mathbb{R}$.

2.18. The Natural Logarithmic Transform [25]:

Emad Kuffi et al. proposed the Natural Logarithmic integral transform in 2019. The integral transform is defined for a function $f(t)$ as:

$\mathrm{IL}\{f(t)\} = F(v) = \int_{t=\frac{1}{v}}^{1} \ln(vt) f(t)dt.$

The basic properties of Natural Logarithmic transformation are:

1) $IL\{t^n\} = \frac{\ln(v)}{n+1} - \frac{\left[1-\left(\frac{1}{v}\right)^{n+1}\right]}{(n+1)^2}, n \neq -1$.
2) $IL\{sin(at)\} = sin(a(ln(v)) - 1) - \frac{1}{v} sin\left(\frac{a}{v}\right) + \frac{sin\left(\frac{a}{v}\right) + av\, sin\left(\frac{a}{v}\right)}{v^2} + \frac{cos(a)+a^2 cos(a)}{a} + \frac{av\, sin(a)+sin(a)}{v} + \frac{cos\left(\frac{a}{v}\right)+a^2 cos\left(\frac{a}{v}\right)}{a}$.

3) $IL\{f^{(n)}(t)\} = \ln(v)\, f^{(n-1)}(1) - \sqrt{\frac{v-1}{3}}\sqrt{\left(f^{(n-1)}(1)\right)^3 - k^3} \quad , n > 0, v > 1$ and $f^{(n)}(1) > k$.
4) $IL\{e^{-at}\} = \ln(v)\left[\frac{1}{v} - \frac{e^{-a}}{a}\right]$.

2.19. Rohit Integral Transform [26]

Rohit Gupta in 2020 proposed an integral transform called the Rohit transform (RT). The integral transform is convergent and defined for a function $f(t)$ as:

$R\{f(t)\} = F(v) = v^3 \int_{t=0}^{\infty} e^{-vt} f(t)dt$, v is a real or complex parameter.

The basic properties of RT transformation are:

1) $R\{t^n\} = \frac{n!}{v^{n-2}}, n = 0,1,2,3, \ldots$
2) $R\{\sin(at)\} = \frac{av^3}{v^2+a^2}$, a is constant, $v > 0$.
3) $R\{\cos(at)\} = \frac{v^4}{v^2+a^2}$, $v > 0$.
4) $R\{\sinh(at)\} = \frac{av^3}{v^2-a^2}, v > |a|$.
5) $R\{\cosh(at)\} = \frac{v^4}{v^2-a^2}$, $v > |a|$.
6) $R\{e^{at}\} = \frac{v^3}{v-a}, v > a$.
7) $R\{f'''(t)\} = v^3F(v) - v^5f(0) - v^4f'(0) - v^3f''(0)$.

2.20. Dinesh Verma Transform (DVT) [27]

In 2020, the Dinesh Verma transform (DVT) was introduced. The transform is convergent and defined for a function $f(t)$ as:

$D\{f(t)\} = v^5 \int_{t=0}^{\infty} e^{-vt} f(t)dt = F(v)$, v is a real or a complex parameter.

The basic properties of DVT are:

1) $D\{t^n\} = \frac{n!}{v^{n-4}}, n = 0,1,2,3, \ldots$
2) $D\{\sin(at)\} = \frac{av^5}{v^2+a^2}$, a is a constant.
3) $D\{\cos(at)\} = \frac{v^6}{v^2+a^2}$.
4) $D\{\sinh(at)\} = \frac{av^5}{v^2-a^2}$.
5) $D\{\cosh(at)\} = \frac{v^6}{v^2-a^2}$.
6) $D\{e^{\mathrm{at}}\} = \frac{v^5}{v-a}$.
7) $D\{f'''(t)\} = v^3F(v) - v^7f(0) - v^6f'(0) - v^5f''(0)$.

2.21. Gupta Transform [28]

In 2020, Rahul Gupta et al. proposed the Gupta transform. The integral transform is convergent and defined for a function $f(t)$ as:

$R_u\{f(t)\} = \frac{1}{v^3}\int_{t=0}^{\infty} e^{-vt} f(t) = F(v), v$ is a real or complex parameter.

The basic properties of Gupta transformation are:

1) $R_u\{t^n\} = \frac{n!}{v^{n+4}}$, $n = 0,1,2,3, \ldots.$
2) $R_u\{\sin(at)\} = \frac{a}{v^3(v^2+a^2)}$, a is a constant, $v > 0$.
3) $R_u\{\cos(at)\} = \frac{1}{v^2(v^2+a^2)}$, $v > 0$.
4) $R_u\{\sinh(at)\} = \frac{a}{v^3(v^2-a^2)}$. $v > |a|$.
5) $R_u\{\cosh(at)\} = \frac{1}{v^2(v^2-a^2)}$, $v > |a|$.
6) $R_u\{e^{at}\} = \frac{1}{v^3(v-a)}$, $v > a$.
7) $R_u\{f''(t)\} = v^2F(v) - \frac{1}{v^2}f(0) - \frac{1}{v^3}f'(0)$.

2.22. SEE Transform [29]

Eman A. Mansour et al. introduced the (Sadiq-Emad-Eman) SEE integral transform in 2021. The integral transform is defined for a function $f(t)$ as:

$S\{f(t)\} = \frac{1}{v^n}\int_{t=0}^{\infty} e^{-vt} f(t)dt = F(v), t \geqslant 0, n \in \mathbb{Z}, L_1 \leqslant v \leqslant L_2 \; and \; L_1, L_2 > 0$.

The basic properties of SEE transformation are:

1) $S\{t^m\} = \frac{m!}{v^{n+m+1}}$, $m = 0,1,2,3, \ldots$
2) $S\{e^{at}\} = \frac{1}{v^n(v-a)}$, a is a constant,
3) $S\{\sin(at)\} = \frac{a}{v^n(v^2+a^2)}$.
4) $S\{\cos(at)\} = \frac{1}{v^{n-1}(v^2+a^2)}$.
5) $S\{\sinh(at)\} = \frac{a}{v^n(v^2-a^2)}$.
6) $\mathrm{S}\{\cosh(\mathrm{at})\} = \frac{1}{v^{n-1}(v^2-a^2)}$.
7) $S\{f^{(m)}(t)\} = \frac{-f^{(m-1)}(0)}{v^n} - \frac{f^{(m-2)}(0)}{v^{n-1}} - \cdots - \frac{f(0)}{v^{n-m+1}} + v^mF(v)$.

2.23. Alenezi Transform [30]

Also, in 2021, the Alenezi integral transform had been proposed by Ahmed M. Alenez. The integral transform is defined for a function $f(t)$ as:

$\rho\{f(t)\} = m(v)\int_{t=0}^{\infty} f(t)e^{-\frac{1}{n(v)}t}dt\ , t \geqslant 0, m(v)$ and $n(v) \neq 0$.

The basic properties of Alemezi transformation are:

1) $\rho\{t^n\} = \frac{\Gamma(n+1)\, m(v)}{(n(v))^{n+1}}\ , n \in \mathbb{R}$.
2) $\rho\{\cos(at)\} = \frac{n(v)m(v)}{n^2(v)+a^2}$, a is a constant.
3) $\rho\{\sin(at)\} = \frac{a\, m(v)}{n^2(v)+a^2}$.
4) $\rho\{e^{at}\} = \frac{m(v)}{n(v)-a}$.
5) $\rho\{f^{(n)}(t)\} = n^n(v)\rho\{f(t)\} - m(v)\sum_{k=0}^{n-1} n^{n-1-k}(v)\, h^{(k)}(0)$.

2.24. Emal-Sara Transform [31]

The Emad-Sara (ES) integral transform was introduced in 2021 by Sara Falih Maktoof et al. The integral transform is defined for a function $f(t)$ as:

$ES\{f(t)\} = F(v) = \frac{1}{v^2}\int_{t=0}^{\infty} e^{-vt}f(t)dt\ , t \geqslant 0, m_1 \leqslant v \leqslant m_2$ and $m_1, m_2 > 0$.

The basic properties of ES transformation are:

1) $ES\{t^n\} = \frac{n!}{v^{n+3}}\ , n = 0,1,2,3, \ldots$.
2) $ES\{e^{at}\} = \frac{1}{v^2(v-a)}$, a is a constant.
3) $ES\{\sin(at)\} = \frac{a}{v^2(v^2+a^2)}$.
4) $ES\{\cos(at)\} = \frac{1}{v(v^2+a^2)}$.
5) $ES\{\sinh(at)\} = \frac{a}{v^2(v^2-a^2)}$.
6) $ES\{\cosh(at)\} = \frac{1}{V(v^2-a^2)}$.
7) $\mathrm{ES}\{f^{(n)}(t)\} = \frac{-f^{(n-1)}(0)}{v^2} + vES\{f^{(n-1)}(t)\}$.

2.25. Complex SEE Integral Transform [32]

The complex (Sadiq-Emad-Eman) SEE transform is another integral transform introduced by Eman A. Mansour in 2021. The integral transform is defined for a function $f(t)$ as:

$S^c\{f(t)\} = F(iv) = \frac{1}{v^n}\int_{t=0}^{\infty} f(t)e^{-ivt}dt\ \ , t \geq 0\ , L_1 \leq v \leq L_2$ and $n \in \mathbb{Z}$.

The basic properties of Complex SEE transformation are:

1) $S^c\{t^m\} = \frac{(-1)^m(i)^{m-1}m!}{v^{n+m+1}}, m = 0,1,2,3, \ldots$
2) $S^c\{e^{at}\} = \frac{1}{v^n}\left\{\frac{a}{a^2+v^2} + i\,\frac{v}{a^2+v^2}\right\}$, a is a constant.
3) $S^c\{\sin(\mathrm{at})\} = \frac{-a}{v^n(v^2-a^2)}$.
4) $S^c\{\cos(\mathrm{at})\} = \frac{-iv}{v^{n(v^2-a^2)}}$.
5) $S^c\{\sinh(at)\} = \frac{-a}{v^n(v^2+a^2)}$.
6) $S^c\{\cosh(at)\} = \frac{-iv}{v^n(v^2+a^2)}$.
7) $S^c\{f^{(m)}(t)\} = \frac{1}{v^n}\{-f^{(m-1)}(0) - ivf^{(m-2)}(0) - (iv)^2 f^{(m-3)}(0) - \cdots - (iv)^{m-1}f_{(0)}] + (iv)^m F(iv)$.

2.26. Soham Transform [33]

In 2021, D.P.Patil et al. introduced a new transform called "Soham transform". The integral transform is defined for a function $f(t)$ as:

$\mathcal{S}[\mathrm{f(t)}] = \mathrm{P(v)} = \frac{1}{\mathrm{v}}\int_0^{\infty} f(t)e^{-v^{\alpha}}tdt\ , \alpha$ is a nonzero real number, $t \geq 0, k1 \leq v \leq k2$

The basic properties of Soham transformation are:

1) $\mathcal{S}[\mathrm{t}] = \frac{1}{v^{2\alpha+1}}$.
2) $\mathcal{S}[\mathrm{t^n}] = \frac{\Gamma(\mathrm{n}+1)}{v^{\alpha n+\alpha+1}}\ , \mathrm{n} = 0,1,2,3, \ldots$
3) $\mathcal{S}[\mathrm{e^{at}}] = \frac{1}{v(v^{\alpha}-a)}$.
4) $\mathcal{S}[\sin(at)] = \frac{a}{v(v^{2\alpha}+a^2)}$.
5) $\mathcal{S}[\cos(at)] = \frac{v^{\alpha}}{v(v^{2\alpha}+a^2)}$.
6) $\mathcal{S}[\sinh(at)] = \frac{av}{v^{2\alpha}-a^2}$.
7) $\mathcal{S}[\cos(at)] = \frac{v^{\alpha}}{v^{2\alpha}-a^2}$.

3. Conclusions

The capability of integral transforms to handle problems from different scientific fields put them in the mathematicians' spotlight. For that reason, since the emergence of the first integral transform (Laplace transform), studies on every aspect of integral transforms have been performed by scientists, scientists studied the properties of some existing transforms and proposed new ones. The proposed integral transforms have some modified or brand-new integral kernel or having different boundaries and sometimes a combination of both cases. From the earliest and most well-known Laplace transform to the most modern transforms, this article covers a variety of integral transforms along with their basic properties.

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