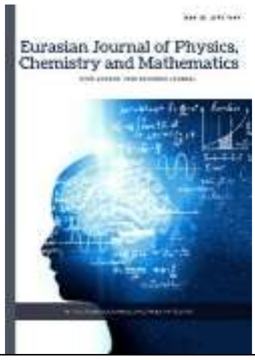

On Tensor ($\hat{C}$) of AK-Manifold

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ABSTRACT

In this study, we examine the Concircular Curvature Tensor ($\hat{C}$) of an almost kahler manifold, which indicates that W2, where W2 stands for the almost kahler manifold, presents one of the almost Hermitian manifold constructions' geometrical characteristics. These results are summarized, with emphasis on the main findings. This tensor's typical Riemannian curvature symmetry features have been proven. Concircular tensor ($\widehat{U}$-tensor) components in the AK- manifold are calculated. gathered some information and built connections between the tensor parts of this manifold. Obtain a reasonable formula for all these components $\widehat{U}0$, $\widehat{U}1$, $\widehat{U}2$, $\widehat{U}3$, $\widehat{U}4$, $\widehat{U}5$, $\widehat{U}6$, $\widehat{U}7$ of almost Kahler manifold

Keywords: Concircular curvature tensor ($\widehat{U}$), almost kahler manifold

Introduction

Differential geometry is characterized by fundamentally Hermitian structures, and Riemannian conformal transformations are a key area of study in this field. The conformal transformation that modifies a geodesic circle does not affect a concircular curvature tensor [13]. The tensor invariance—also known as tensor concircular curvature is a property of these transformations. In 1975, the Russian researcher Kirichenko developed a novel strategy for examining the various classes of nearly Hermitian manifolds. This strategy is based on M s.t. U, a unitary group with a structure (n). An adjoined G-structure space is what is meant by this. Two tensors, the structure and virtual tensors, were established by Russian researcher Kirichenko after he researched a nearly Hermitian manifold in detail using adjoined G-structure space [9]. He was able to find the structure group of a nearly Hermitian manifold with the help of these tensors. Born in 1993, Banaru [2] After the person who was successful in reclassifying the classes 16 of nearly Hermitian manifolds using virtual tensors and structure, Kirichenko's tensors were named [8].In 2015 saw the division of the "concircular curvature tensor of NK-manifold" search into three parts by Rawah. A. Z. Kahler and almost kahler manifolds were examined in the first section. It was explained what kahler and almost were as well as how they related to one another. The basics of alternating and structural tensors are covered in the second section. The NK-manifold circular curvature tensor is the focus of the third section. [10]. The "The Concircular Curvature Tensor of Viashman, Gray 1 manifold" search was divided into three parts by Ebtihal Q. Ali in 2020. We shall examine the elements of the VG-structure manifold's equation in the first section. The essential ideas of alternating and structural tensors will be covered in the second section. Finally, utilizing the notion of holomorphic sectional curvature tensor (BHC-cur [5], the VG-manifold of appointwise holomorphic sectional concircular curvature tensor (PHC-tensor) has been investigated. Majed investigated the Conharmonic curvature tensors of a nearly Kahler manifold and a pointwise holomorphic sectional curvature manifold [8].We look into both the AK-concircular manifold's curvature tensor and the class of almost kahler manifolds.

Preliminaries :

Let the basic form Ω (τ, μ) = <τ, Jμ> is closed i. e d Ω = 0, The Hermitian manifold is then stated to have roughly< M, J, g =.,.> and have an almost kahler structure (AK structure). A smooth M manifold with an AK chassis is known as a roughly kahler manifold (AK-manifold). Additionally, we assert that instances of a smooth function f- $C^{\wedge\infty}(M)$ s.t $\tilde{g} = e^{2f}g$ on M given the matching shunt scale, provided g and $\tilde{g}$ are two Riemannian scales on the smooth manifold M.

Theorem 1: [7]

The following are the components of the adjoint G-structure space Riemannian curvature tensor of the AK-manifold :

1) $R^a{}_{bcd} = 2B^a{}_{bcd}$.
2) $R^a{}_{\hat{b}cd} = 4B^{hab} B_{hcd}$.
3) $R^a{}_{b\hat{c}d} = 4B^{cah} B_{dbh} - A^{ac}{}_{bd} - 2B^{ach} B_{hbd}$.
4) $R^a{}_{bc\hat{d}} = A^{ad}{}_{bc} + 2B^{adh} B_{hbc} - 4B^{dah} B_{cbh}$.
5) $R^a{}_{b\hat{c}\hat{d}} = 2B^{adc}{}_b$.
6) $R^a{}_{\hat{b}\hat{c}d} = -2B^{cab}{}_d$
7) $R^a{}_{\hat{b}c\hat{d}} = 2B^{dab}{}_c$
8) $R^a{}_{\hat{b}\hat{c}\hat{d}} = -4B^{[c|ab|d]}$
9) $R^{\hat{a}}{}_{bcd} = -4B_{[c|ab|d]}$
10) $R^{\hat{a}}{}_{\hat{b}cd} = -2B^b{}_{acd}$
11) $R^{\hat{a}}{}_{b\hat{c}d} = 2B^c{}_{dab}$.
12) $R^{\hat{a}}{}_{bc\hat{d}} = -2B^d{}_{cab}$.
13) $R^{\hat{a}}{}_{b\hat{c}\hat{d}} = 4B^{hcd} B_{hab}$.
14) $R^{\hat{a}}{}_{\hat{b}\hat{c}d} = A^{bc}{}_{ad} + 2B^{bch} B_{had} - 4B^{dah} B_{cbh}$.
15) $R^{\hat{a}}{}_{\hat{b}c\hat{d}} = 4B^{dbh} B_{cah} - A^{bd}{}_{ac} - 2B^{bdh} B_{hac}$.
16) $R^{\hat{a}}{}_{\hat{b}\hat{c}\hat{d}} = -B^{bcd}{}_a$.

Definition 2: [9]

A tensor of type (2,0) is said to be Ricci tensor and defined by the form.: $r_{ij} = R^k{}_{ijk} = g^{ki} R_{kijl}$

Definition 3: [9]

A scalar curvature tensor denoted by k which is defined by: $k = g^{ij} r_{ij}$

Theorem 4: [2]

A components of the Ricci tensor for the AK-manifold are represented by the following figures in the adjacent structure space G:

1) $r_{ab} = 4B^c{}_{(ab)c}$
2) $r_{\hat{a}\hat{b}} = 4B^{(ab)c}{}_c$.
3) $r_{\hat{a}b} = -4B^{hac} B_{hcb} - A^{ac}{}_{cb} - 2B^{ach} B_{hcb} + 4B^{cah} B_{bch}$.
4) $r_{a\hat{b}} = -4B^{hcb} B_{hca} - A^{bc}{}_{ca} - 2B^{bch} B_{hca} + 4B^{cbh} B_{ach}$.

Remark 5: [4]

1) The notation g is used to express the value of the Riemannian scale:
i) $g_{ab} = g_{\hat{a}\hat{b}} = 0$.
iii) $g_{\hat{a}b} = \delta^{a}{}_{b}$.
iv) $g_{a\hat{b}} = \delta^{b}{}_{a}$.

Definition 6 [3]

The form of the Covariant Concircular Curvatare ($\hat{C}$) of type (3,1) of AK-manifold is

$\widehat{U}(\omega_1, \omega_2)\,\omega_3 = R(\omega_1, \omega_2)\,\omega_3 - \frac{s}{2n(2n+1)}\{g(\omega_2, \omega_3)\,\omega_1 - g(\omega_1, \omega_3)\,\omega_2\}$ (1)

The form of the Contravariant Concircular Curvatare ($\hat{C}$) of type (4,0) of AK-manifold is

$\widehat{U}(\omega_1, \omega_2, \omega_3, \omega_4) = R(\omega_1, \omega_2, \omega_3, \omega_4) = -\frac{s}{2n(2n+1)}\{(\omega_1, \omega_4)\, g(\omega_1, \omega_2) - r(\omega_2, \omega_4)\, g(\omega_1, \omega_3)\}$ (2)

Where s is the scalar curvature, g is the Riemannian metric, r represents the Ricci tensor, and R is the Riemannian curvature tensor s.t $\omega_1, \omega_2, \omega_3, \omega_4 \in X(M)$

Properties 7

As result , the annular bending tensor ($\widehat{U}$) of AK-manifold meets all of the algebraic bending tensor's requirements:

1) $\widehat{U}(\omega_1, \omega_2, \omega_3, \omega_4) = -\widehat{U}(\omega_2, \omega_1, \omega_3, \omega_4)$
2) $\widehat{U}(\omega_1, \omega_2, \omega_3, \omega_4) = -\widehat{U}(\omega_1, \omega_2, \omega_4, \omega_3)$;
3) $\widehat{U}(\omega_1, \omega_2, \omega_3, \omega_4) + \widehat{U}(\omega_2, \omega_3, \omega_1, \omega_4) + \widehat{U}(\omega_4, \omega_1, \omega_2, \omega_3) = 0$;
4) $\widehat{U}(\omega_1, \omega_2, \omega_3, \omega_4) = \widehat{U}(\omega_3, \omega_4, \omega_1, \omega_2)$ $\omega_1, \omega_2, \omega_3, \omega_4 \in X(M)$. (3)

Proof: - we shall prove (1)

1) $\widehat{U}(\omega_1, \omega_2, \omega_3, \omega_4) = R(\omega_1, \omega_2, \omega_3, \omega_4) - \frac{s}{2n(2n+1)}\{(\omega_1, \omega_4)\, g(\omega_1, \omega_2) - r(\omega_2, \omega_4)\, g(\omega_1, \omega_3)\}$

$= -R(\omega_2, \omega_1, \omega_3, \omega_4) + \frac{s}{2n(2n+1)}\{(\omega_1, \omega_4)\, g(\omega_1, \omega_2) - r(\omega_2, \omega_4)\, g(\omega_1, \omega_3)\}$

$= -\widehat{U}(\omega_2, \omega_1, \omega_3, \omega_4)$

Properties are proven in a similar way :

2) $\widehat{U}(\omega_1, \omega_2, \omega_3, \omega_4) = -\widehat{U}(\omega_1, \omega_2, \omega_4, \omega_3)$;
3) $\widehat{U}(\omega_1, \omega_2, \omega_3, \omega_4) + \widehat{U}(\omega_2, \omega_3, \omega_1, \omega_4) + \widehat{U}(\omega_4, \omega_1, \omega_2, \omega_3) = 0$;
4) $\widehat{U}(\omega_1, \omega_2, \omega_3, \omega_4) = \widehat{U}(\omega_3, \omega_4, \omega_1, \omega_2)$; $\omega_1, \omega_2, \omega_3, \omega_4 \in X(M)$

Remark 8:

The following form can be written using the definition of a spectrum tensor:

$\widehat{U}(\omega_1, \omega_2)\,\omega_3 = \widehat{U}_0(\omega_1, \omega_2)\,\omega_3 + \widehat{U}_1(\omega_1, \omega_2)\,\omega_3 + \widehat{U}_2(\omega_1, \omega_2)\,\omega_3 + \widehat{U}_3(\omega_1, \omega_2)\,\omega_3 + \widehat{U}_4(\omega_1, \omega_2)\,\omega_3 + \widehat{U}_5(\omega_1, \omega_2)\,\omega_3 + \widehat{U}_6(\omega_1, \omega_2)\,\omega_3 + \widehat{U}_7(\omega_1, \omega_2)\,\omega_3$; $\omega_1, \omega_2, \omega_3, \omega_4 \in X(M)$.

tensor $\widehat{U}_0(\omega_1, \omega_2)\,\omega_3$ as a non-zero component containing just the model's components $\{\widehat{U}_0{}^{a}{}_{bcd}, \widehat{U}_0{}^{\hat{a}}{}_{\hat{b}\hat{c}\hat{d}}\} = \{\widehat{U}^{a}{}_{bcd}, \widehat{U}^{\hat{a}}{}_{\hat{b}\hat{c}\hat{d}}\}$;

tensor $\widehat{U}_1(\omega_1, \omega_2)\,\omega_3$ - components of the form $\{\widehat{U}_1{}^{a}{}_{bc\hat{d}}, \widehat{U}_1{}^{\hat{a}}{}_{\hat{b}\hat{c}d}\} = \{\widehat{U}^{a}{}_{bc\hat{d}}, \widehat{U}^{\hat{a}}{}_{\hat{b}\hat{c}d}\}$;

tensor $\widehat{U}_2(\omega_1, \omega_2)\,\omega_3$- components of the form $\{\widehat{U}_2{}^{a}{}_{b\hat{c}d}, \widehat{U}_2{}^{\hat{a}}{}_{\hat{b}c\hat{d}}\} = \{\widehat{U}^{a}{}_{b\hat{c}d}, \widehat{U}^{\hat{a}}{}_{\hat{b}c\hat{d}}\}$;

tensor $\widehat{U}_3(\omega_1, \omega_2)\,\omega_3$ - components of the form $\{\widehat{U}_3{}^{a}{}_{b\hat{c}\hat{d}}, \widehat{U}_3{}^{\hat{a}}{}_{\hat{b}cd}\} = \{\widehat{U}^{a}{}_{b\hat{c}\hat{d}}, U\}$;

tensor $\widehat{U}_4(\omega_1, \omega_2)\,\omega_3$- components of the form $\{\widehat{U}_4{}^{a}{}_{\hat{b}cd}, \widehat{U}_4{}^{\hat{a}}{}_{b\hat{c}\hat{d}}\} = \{\widehat{U}^{a}{}_{\hat{b}cd}, \widehat{U}^{\hat{a}}{}_{b\hat{c}\hat{d}}\}$;

tensor $\widehat{U}_5(\omega_1, \omega_2)\,\omega_3$- components of the form $\{\widehat{U}_5{}^{a}{}_{\hat{b}c\hat{d}}, \widehat{U}_5{}^{\hat{a}}{}_{b\hat{c}d}\} = \{\widehat{U}^{a}{}_{\hat{b}c\hat{d}}, \widehat{U}^{\hat{a}}{}_{b\hat{c}d}\}$;

tensor $\widehat{U}_6(\omega_1, \omega_2)\,\omega_3$ - components of the form $\{\widehat{U}_6{}^{a}{}_{\hat{b}\hat{c}d}, \widehat{U}_6{}^{\hat{a}}{}_{bc\hat{d}}\} = \{\widehat{U}_6{}^{a}{}_{\hat{b}\hat{c}d}, \widehat{U}_6{}^{\hat{a}}{}_{bc\hat{d}}\}$;

tensor $\widehat{U}_7$ (ω_1, ω_2) ω_3- components of the form $\{\widehat{U}_7^a{}_{\hat{b}\hat{c}\hat{d}}, \widehat{U}_7^{\hat{a}}{}_{bcd}\} = \{\widehat{U}^a{}_{\hat{b}\hat{c}\hat{d}}, \widehat{U}^{\hat{a}}{}_{bcd}\}$.

Tensors $\widehat{U}_0 = \widehat{U}_0$ (ω_1, ω_2) ω_3 , $\widehat{U}_1 = \widehat{U}_1$ (ω_1, ω_2) ω_3 , ..., $\widehat{U}_7 = \widehat{U}_7$ (ω_1, ω_2) ω_3

The fundamental invariants the Concircular ($\widehat{C}$) AH-manifold will be given a name.

Definition 9

According to the definition given above, we find an AK-manifold with $\widehat{U}_i$ = 0 is represents an AK-manifold of class $\widehat{U}_i$ where i = 0, 1,..., 7.

Theorem 10

1) AK − manifold of class $\widehat{U}_0$ (AK) distinguished by identity
$\widehat{U}(\omega_1, \omega_2)\,\omega_3 - \widehat{U}(\omega_1, J\omega_2)\,J\omega_3 - \widehat{U}(J\omega_1, \omega_2)\,J\omega_3 - \widehat{U}(J\omega_1, J\omega_2)\,\omega_3 - J\widehat{U}(\omega_1, \omega_2)\,J\omega_3 - J\widehat{U}(\omega_1, J\omega_2)\,\omega_3 - J\widehat{U}(J\omega_1, \omega_2)\,\omega_3 + J\widehat{U}(J\omega_1, J\omega_2)\,J\omega_3 = 0$; $\omega_1, \omega_2, \omega_3 \in X(M)$. (4)

2) AK - manifold of class $\widehat{U}_1(AK)$ described by identity
$\widehat{U}(\omega_1, \omega_2)\,\omega_3 + \widehat{U}(\omega_1, J\omega_2)\,J\omega_3 - \widehat{U}(J\omega_1, \omega_2)\,J\omega_3 + \widehat{U}(J\omega_1, J\omega_2)\,\omega_3 + J\widehat{U}(\omega_1, \omega_2)\,J\omega_3 - J\widehat{U}(\omega_1, J\omega_2)\,\omega_3 - J\widehat{U}(J\omega_1, \omega_2)\,\omega_3 - J\widehat{U}(J\omega_1, J\omega_2)\,J\omega_3 = 0$; $\omega_1, \omega_2, \omega_3 \in X(M)$. (5)

3) AK - manifold of class $\widehat{U}_2(AK)$ described by identity
$\widehat{U}(\omega_1, \omega_2)\,\omega_3 - \widehat{U}(\omega_1, J\omega_2)\,J\omega_3 + \widehat{U}(J\omega_1, \omega_2)\,J\omega_3 + \widehat{U}(J\omega_1, J\omega_2)\,\omega_3 - J\widehat{U}(\omega_1, \omega_2)\,J\omega_3 - J\widehat{U}(\omega_1, J\omega_2)\,\omega_3 + J\widehat{U}(J\omega_1, \omega_2)\,\omega_3 - J\widehat{U}(J\omega_1, J\omega_2)\,J\omega_3 = 0$; $\omega_1, \omega_2, \omega_3 \in X(M)$. (6)

4) AK - manifold of class $\widehat{U}_3(AK)$ described by identity
$\widehat{U}(\omega_1, \omega_2)\,\omega_3 + \widehat{U}(\omega_1, J\omega_2)\,J\omega_3 + \widehat{U}(J\omega_1, \omega_2)\,J\omega_3 - \widehat{U}(J\omega_1, J\omega_2)\,\omega_3 - J\widehat{U}(\omega_1, \omega_2)\,J\omega_3 + J\widehat{U}(\omega_1, J\omega_2)\,\omega_3 + J\widehat{U}(J\omega_1, \omega_2)\,\omega_3 + J\widehat{U}(J\omega_1, J\omega_2)\,J\omega_3 = 0$; $\omega_1, \omega_2, \omega_3 \in X(M)$. (7)

5) AK- manifold of class $\widehat{U}_4(AK)$ described by identity
$\widehat{U}(\omega_1, \omega_2)\,\omega_3 + \widehat{U}(\omega_1, J\omega_2)\,J\omega_3 + \widehat{U}(J\omega_1, \omega_2)\,J\omega_3 - \widehat{U}(J\omega_1, J\omega_2)\,\omega_3 + J\widehat{U}(\omega_1, \omega_2)\,J\omega_3 - J\widehat{U}(\omega_1, J\omega_2)\,\omega_3 - J\widehat{U}(J\omega_1, \omega_2)\,\omega_3 - J\widehat{U}(J\omega_1, J\omega_2)\,J\omega_3 = 0$; $\omega_1, \omega_2, \omega_3 \in X(M)$. (8)

6) AK - manifold of class $\widehat{U}_5(AK)$ described by identity
$\widehat{U}(\omega_1, \omega_2)\,\omega_3 - \widehat{U}(\omega_1, J\omega_2)\,J\omega_3 + \widehat{U}(J\omega_1, \omega_2)\,J\omega_3 + \widehat{U}(J\omega_1, J\omega_2)\,\omega_3 + J\widehat{U}(\omega_1, \omega_2)\,J\omega_3 + J\widehat{U}(\omega_1, J\omega_2)\,\omega_3 - J\widehat{U}(J\omega_1, \omega_2)\,\omega_3 + J\widehat{U}(J\omega_1, J\omega_2)\,J\omega_3 = 0$; $\omega_1, \omega_2, \omega_3 \in X(M)$. (9)

7) AK- manifold of class $\widehat{U}_6(AK)$ described by identity
$\widehat{U}(\omega_1, \omega_2)\,\omega_3 + \widehat{U}(\omega_1, J\omega_2)\,J\omega_3 - \widehat{U}(J\omega_1, \omega_2)\,J\omega_3 + \widehat{U}(J\omega_1, J\omega_2)\,\omega_3 + J\widehat{U}(\omega_1, \omega_2)\,J\omega_3 - J\widehat{U}(\omega_1, J\omega_2)\,\omega_3 + J\widehat{U}(J\omega_1, \omega_2)\,\omega_3 + J\widehat{U}(J\omega_1, J\omega_2)\,J\omega_3 = 0$; $\omega_1, \omega_2, \omega_3 \in X(M)$. (10)

8) AK - manifold of class $\widehat{U}_7(AK)$ described by identity
$\widehat{U}(\omega_1, \omega_2)\,\omega_3 - \widehat{U}(\omega_1, J\omega_2)\,J\omega_3 - \widehat{U}(J\omega_1, \omega_2)\,J\omega_3 - \widehat{U}(J-\sqrt{-1}, J\omega_2)\,\omega_3 + J\widehat{U}(\omega_1, \omega_2)\,J\omega_3 + J\widehat{U}(\omega_1, J\omega_2)\,\omega_3 + J\widehat{U}(J\omega_1, \omega_2)\,\omega_3 - J\widehat{U}(J\omega_1, J\omega_2)\,J\omega_3 = 0$; $\omega_1, \omega_2, \omega_3 \in X(M)$ (11)

Proof: -

1) Let AK- manifold of class $\widehat{U}_0$ (AK), the manifold of class $\widehat{U}_0(AK)$ distinguished by a condition :
$\widehat{U}_0{}^a{}_{bcd} = 0$, or $\widehat{U}^a{}_{bcd} = 0$. i.e., $\left[\widehat{U}(\varepsilon_c, \varepsilon_d)\varepsilon_b\right]^a \varepsilon_a$.

As σ - a projector on $D_J^{\sqrt{-1}}$,that $\sigma \circ \{\widehat{U}(\sigma\,\omega_1, \sigma\,\omega_2)\,\sigma\omega_3\} = 0$; i. e (id-$\sqrt{-1}$J){ $\widehat{U}$ ($\omega_1 - \sqrt{-1}J\omega_1$, $\omega_2 - \sqrt{-1}J\omega_2$, $\omega_3 - \sqrt{-1}J\omega_3$)}=0

It is possible to remove the brackets.: i.e $\widehat{U}(\omega_1, \omega_2)\,\omega_3 - \widehat{U}(\omega_1, J\omega_2)\,T\omega_3 - \widehat{U}(J\omega_1, \omega_2)\,J\omega_3 - \widehat{U}(J\omega_1, J\omega_2)\,\omega_3 - J\widehat{U}(\omega_1, \omega_2)\,J\omega_3 - J\widehat{U}(\omega_1, J\omega_2)\,\omega_3 - J\widehat{U}(J\omega_1, \omega_2)\,\omega_3 + J\widehat{U}(J\omega_1, J\omega_2)\,J\omega_3 - \sqrt{-1}\,\{\widehat{U}(\omega_1, \omega_2)J\,\omega_3 + \widehat{U}(\omega_1, J\omega_2)\,\omega_3 + \widehat{U}(J\omega_1, \omega_2)\,\omega_3 - \widehat{U}(J\omega_1, J\omega_2)\,J\omega_3 - J\widehat{U}(\omega_1, \omega_2)\,\omega_3 - J\widehat{U}(\omega_1, J\omega_2)\,J\omega_3 - J\widehat{U}(J\omega_1, \omega_2)\,J\omega_3 - J\widehat{U}(J\omega_1, J\omega_2)\,\omega_3\} = 0$. i.e

1) $\widehat{U}(\omega_1, \omega_2)\,\omega_3 - \widehat{U}(\omega_1, J\omega_2)\,J\omega_3 - \widehat{U}(J\omega_1, \omega_2)\,J\omega_3 - \widehat{U}(J\omega_1, J\omega_2)\,\omega_3 - J\widehat{U}(\omega_1, \omega_2)\,J\omega_3 - J\widehat{U}(\omega_1, J\omega_2)\,\omega_3 - J\widehat{U}(J\omega_1, \omega_2)\,\omega_3 + J\widehat{U}(J\omega_1, J\omega_2)\,J\omega_3 = 0$ ……………(12)

2) $\widehat{U}$(ω1, ω2) Jω3 + $\widehat{U}$(ω1, Jω2) ω3 + $\widehat{U}$(Jω1, ω2) ω3 - $\widehat{U}$(Jω1, Jω2) Jω3 +J$\widehat{U}$(ω1, ω2) ω3 -$J\widehat{U}$(ω1, Jω2) Jω3 -J $\widehat{U}$(Jω1, ω2) Jω3 -$J\widehat{U}$(Jω1, Jω2) Jω3 =0 …………..(13)

These equalities (12) and (13) can be interchanged the initial substitution results in the second equality. ω3 on Jω3

Thus AK - manifold of class $\widehat{U}_0(AK)$ characterized by identity

$\widehat{U}$(ω1, ω2) ω3 - $\widehat{U}$(ω1, Jω2) Tω3 - $\widehat{U}$(Jω1, ω2) Jω3 - $\widehat{U}$(Jω1, Jω2) ω3 -J$\widehat{U}$(ω1, ω2) Jω3 -$J\widehat{U}$(ω1, Jω2) ω3 -J $\widehat{U}$(Jω1, ω2) ω3 +$J\widehat{U}$(Jω1, Jω2) Jω3 =0 ;ω1, ω2 ,ω3 $\in X(M)$ …….. …. (14)

Assume of AK as having a wide variety of classes. The 2,3,4,5,6,7, and 8 can be received from U_1(AK) to U_7 (AK).

Theorem 11

The following are the components of the AK- manifold's tensor $(\widehat{U})$ in the adjont G-structure space :

1) $\widehat{U}_{abcd} = -4B_{[c|ab|d]}$

2) $\widehat{U}_{\hat{a}bcd} = 2B^{a}_{bcd} + \frac{s}{2n(2n+1)}\{4B^{m}_{(bd)m}\,\delta^{a}_{c}\}$

3) $\widehat{U}_{a\hat{b}cd} = -2B^{b}_{acd}$

4) $\widehat{U}_{ab\hat{c}d} = 2B^{c}_{dab} - \frac{s}{2n(2n+1)}\{4B^{m}_{(ad)m}\,\delta^{c}_{b} - 4B^{m}_{(bd)m}\delta^{c}_{a}\}$

5) $\widehat{U}_{abc\hat{d}} = 2B^{d}_{cab}$

6) $\widehat{U}_{\hat{a}\hat{b}cd} = 4B^{hab}B_{hcd} - \frac{s}{2n(2n+1)}\{r_{\hat{a}d}\,\delta^{b}_{c} - r_{\hat{b}d}\,\delta^{b}_{d}\}$

7) $\widehat{U}_{\hat{a}b\hat{c}d} = 4B^{cah}\,B_{dbh} - A^{ac}_{bd} - 2B^{ach}\,B_{cbh} - \frac{s}{2n(2n+1)}\{r_{\hat{a}d}\delta^{c}_{b}\}$

8) $\widehat{U}_{\hat{a}bc\hat{d}} = A^{ad}_{bc} + 2B^{adh}\,B_{hbc} - 4B^{dah}\,B_{cbh} + \frac{s}{2n(2n+1)}\{r_{b\hat{a}}\,\delta^{a}_{c}\}$

Proof:

1) Put i = a , j = b , k = c, l =d .

$\widehat{U}_{abcd} = R_{abcd} - \frac{s}{2n(2n+1)}\{r_{ad}\ g_{bc} - r_{bd}\ g_{ac}\}$

$\widehat{U}_{abcd} = R_{abcd} - \frac{s}{2n(2n+1)}\{r_{ad}\ (0) - r_{bd}\ (0)\}$

$\widehat{U}_{abcd} = R_{abcd} = -4B_{[c|ab|d]}$

2)Put i = a^ , j = b , k = c , l = d .

$\widehat{U}_{\hat{a}bcd} = R_{\hat{a}bcd} - \frac{s}{2n(2n+1)}\{r_{\hat{a}d}\ g_{bc} - r_{bd}\ g_{\hat{a}c}\}$

$\widehat{U}_{\hat{a}bcd} = R_{\hat{a}bcd} - \frac{s}{2n(2n+1)}\{r_{\hat{a}d}\ (0) - r_{bd}\ g_{\hat{a}c}\}$

$\widehat{U}_{\hat{a}bcd} = R_{\hat{a}bcd} + \frac{s}{2n(2n+1)}\{r_{bd}\,g_{\hat{a}c}\}$

$\widehat{U}_{\hat{a}bcd} = 2B^{a}_{bcd} + \frac{s}{2n(2n+1)}\{4B^{m}_{(bd)m}\,\delta^{a}_{c}\}$

3)Put i = a , j = b^ , k = c , l = d

$\widehat{U}_{a\hat{b}cd} = R_{a\hat{b}cd} - \frac{s}{2n(2n+1)}\{r_{ad}\ g_{\hat{b}c} - r_{\hat{b}d}\ g_{ac}\}$

$\widehat{U}_{a\hat{b}cd} = R_{a\hat{b}cd} - \frac{s}{2n(2n+1)}\{r_{ad}\ (0) - r_{\hat{b}d}\ (0)\}$

$\widehat{U}_{a\hat{b}cd} = R_{a\hat{b}cd} = -2B^{b}_{acd}$

4)Put i = a , j = b , k=c^ , l =d

$\widehat{U}_{ab\hat{c}d} = R_{ab\hat{c}d} - \frac{s}{2n(2n+1)}\{r_{ad}\ g_{b\hat{c}} - r_{bd}\,g_{a\hat{c}}\}$

$\widehat{U}_{ab\hat{c}d} = 2B^{c}_{dab} - \frac{s}{2n(2n+1)}\{4B^{m}_{(ad)m}\,\delta^{c}_{b} - 4B^{m}_{(bd)m}\delta^{c}_{a}\}$

5)Put i = a , j=b , k = c , l = d^

$\widehat{U}_{abc\hat{d}} = R_{abc\hat{d}} - \frac{s}{2n(2n+1)}\{r_{a\hat{d}}\ g_{bc} - r_{b\hat{d}}\ g_{ac}\}$

$\widehat{U}_{abc\hat{d}} = R_{abc\hat{d}} - \frac{s}{2n(2n+1)}\{r_{a\hat{d}}\ (0) - r_{b\hat{d}}\ (0)\}$

$\widehat{U}_{abc\hat{d}} = R_{abc\hat{d}} = -2B^{d}_{cab}$

6)put i = a^ , j = b^ , k = c , l = d

$\widehat{U}_{\hat{a}\hat{b}cd} = R_{\hat{a}\hat{b}cd} - \frac{s}{2n(2n+1)}\{r_{\hat{a}d}\ g_{\hat{b}c} - r_{\hat{b}d}\ g_{\hat{a}c}\}$

$\widehat{U}_{\hat{a}\hat{b}cd} = 4B^{hab}B_{hcd} - \frac{s}{2n(2n+1)}\{r_{\hat{a}d}\ \delta^{b}_{c} - r_{\hat{b}d}\ \delta^{b}_{d}\}$

7)Put i = a^ , j = b , k = c^ , l = d

$\widehat{U}_{\hat{a}b\hat{c}d} = R_{\hat{a}b\hat{c}d} - \frac{s}{2n(2n+1)}\{r_{\hat{a}d}\ g_{b\hat{c}} - r_{bd}\ g_{\hat{a}\hat{c}}\}$

$\widehat{U}_{\hat{a}b\hat{c}d} = R_{\hat{a}b\hat{c}d} - \frac{s}{2n(2n+1)}\{r_{\hat{a}d}\ g_{b\hat{c}} - r_{bd}\ (0)\}$

$\widehat{U}_{\hat{a}b\hat{c}d} = 4B^{cah}\ B_{dbh} - A^{ac}_{bd} - 2B^{ach}\ B_{cbh} - \frac{s}{2n(2n+1)}\{r_{\hat{a}d}\delta^{c}_{b}\}$

8)Put i = a^ , j=b , k = c , l = d^

$\widehat{U}_{\hat{a}bc\hat{d}} = R_{\hat{a}bc\hat{d}} - \frac{s}{2n(2n+1)}\{r_{\hat{a}\hat{d}}\ g_{bc} - r_{b\hat{d}}\ g_{\hat{a}c}\}$

$\widehat{U}_{\hat{a}bc\hat{d}} = R_{\hat{a}bc\hat{d}} - \frac{s}{2n(2n+1)}\{r_{\hat{a}\hat{d}}\ (0) - r_{b\hat{d}}\ g_{\hat{a}c}\}$

$\widehat{U}_{\hat{a}bc\hat{d}} = A^{ad}_{bc} + 2B^{adh}\ B_{hbc} - 4B^{dah}\ B_{cbh} + \frac{s}{2n(2n+1)}\{r_{b\hat{d}}\ \delta^{a}_{c}\}$

Remark 12

From above theorem (3.3.7) , we can write above by the following forms :

1) $\widehat{U}_0 = -4B_{[c|ab|d]}$

2) $\widehat{U}_1 = 2B^{a}_{bcd} + \frac{s}{2n(2n+1)}\{4B^{m}_{(bd)m}\ \delta^{a}_{c}\}$

3) $\widehat{U}_2 = -2B^{b}_{acd}$

4) $\widehat{U}_3 = 2B^{c}_{dab} - \frac{s}{2n(2n+1)}\{4B^{m}_{(ad)m}\ \delta^{c}_{b} - 4B^{m}_{(bd)m}\delta^{c}_{a}\}$

5) $\widehat{U}_4 = 2B^{d}_{cab}$

6) $\widehat{U}_5 = 4B^{hab}B_{hcd} - \frac{s}{2n(2n+1)}\{r_{\hat{a}d}\ \delta^{b}_{c} - r_{\hat{b}d}\ \delta^{b}_{d}\}$

7) $\widehat{U}_6 = 4B^{cah}\ B_{dbh} - A^{ac}_{bd} - 2B^{ach}\ B_{cbh} - \frac{s}{2n(2n+1)}\{r_{\hat{a}d}\delta^{c}_{b}\}$

8) $\widehat{U}_7 = A^{ad}_{bc} + 2B^{adh}\ B_{hbc} - 4B^{dah}\ B_{cbh} + \frac{s}{2n(2n+1)}\{r_{b\hat{d}}\ \delta^{a}_{c}\}$

Conclusions

After studying this topic, we found that:
The notion of tensor ($\widehat{U}$) concircular curvature has been studied and analyzing and Computing of the components of $\widehat{U}$ of Ak - manifold and we found all components of curvature tensor do not equal Zero.

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