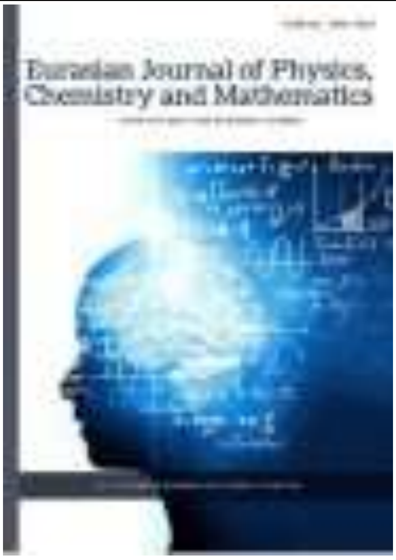

Perturbation due to third body on Sun-synchronous satellite during a long period

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ABSTRACT

A sun-synchronous orbit, sometimes called an helio-synchronous orbit, is when the Earth orbits the sun at a constant angle relative to the Earth-sun direction. In this work, the analytical technique for third body perturbation on sun-synchronous orbit satellites for different values of Node is considered computed for long period of time At a time interval of 180 day, the dynamic development of sun-synchronous orbits is considered. It was accomplished by utilizing the numerical output results from the celestial mechanics' version 1 software program package. Numerical motion simulations were performed and The integration was carried out by using the Celestial Mechanics software program SATORB module (Beutler, 2005) created at the University of Bern's Institute of Astronomy. With input data given by the Two-Line Elements (TLE). Represented by six orbital elements and three Coordinates axes and acceleration components which were used to solve the variation of parameters equations (VOP) using a technique known as collocation method. It is reasonable to assume that the change in orbital elements with the greatest Mean effect on. With the addition of the Geocentric equatorial coordinate system and acceleration combinations. The results show that the third body perturbation has the greatest influence on orbital elements for perigee and Mean anomaly sections and acceleration components with a small influence caused by third body for Geocentric equatorial coordinates sections with increasing and decreasing rate of change as secular and periodic effects and that the perturbation is amplified by satellite heights and mean altitude.

Keywords: Gravitational perturbation effects, Celestial mechanics, Keplerian elements, Sun-synchronous orbit.

1. Introduction

In astrodynamics, studying and modeling perturbations are important fields. Even though the majority of the solution approaches have been around for a long time, Perturbations are motions that deviate from a normal, idealized, or unaltered state. We tend to think of the cosmos as being quite regular and predictable. However, good observational data frequently exposes inexplicable anomalies of motion overlaid on the celestial bodies' more regular or mean movements [1]. Although the exact location of a low Earth orbit is unclear, it is generally thought to be between 100 and 1000 kilometers above the Earth's surface. This is the most cost-effective and straightforward orbit for a spacecraft to enter. Getting a spacecraft into a low-earth orbit (LEO) uses less energy than getting it into a higher-altitude orbit [2]. Due to the impact of numerous "disturbing" factors, a satellite's real orbit deviates from the Keplerian orbit. This includes, among other things, the Earth's non-spherical gravity, the gravitational influences of the moon and sun, atmospheric air drag, and solar radiation pressure. These disrupting forces produce temporal changes in the orbital elements of secular, long- and short-periodic nature (orbital perturbations). The real orbit can be thought of as the envelope of Keplerian ellipses provided by the actual orbital components at any given time (osculating ellipses). Artificial Earth satellites have been used for geodetic applications such as locating and determining the Earth's gravitational field and rotation characteristics since the launch of Sputnik I in 1957. Only a few satellite missions have been specifically intended for geodetic purposes. However, geodesy makes considerable use of a vast number of satellites produced for navigation, remote sensing, and geophysics [3]. Satellites are attracted not just by the Earth's central force, but also by its non-central force, the sun and moon's attractive forces, and the drag force of the atmosphere. Solar radiation pressure, Earth and ocean tides, general relativity effects, and coordinate perturbations all have an impact on them. Satellite motion equations must be expressed using perturbed equations [4]. Third body perturbation has a long time of study and investigations with many research and papers concluding different subjects working with this topic these as an example for this study is Kozai Yoshihide (1959), the current article deduces as a function of mean orbital elements and time perturbations of six orbital elements of a near-earth satellite passing through the earth's gravitational field without meeting air resistance. No assumptions are made about the degree of eccentricity or inclination [5]. According to Kozai Yoshihide (1973) this paper pioneered an entirely new approach for computing lunisolar perturbations. The disturbing function is defined as the satellite's orbital components and the sun and moon's polar coordinates [6]. Lara M. (2012) used perturbation theory based on Lie transformations and higher-order averaging to investigate the long-term evolution of GNSS-type orbits[7]. Roscoe W.T.C. (2015) studied the impact of lunisolar perturbation on a satellite by utilizing differential orbital elements to express the relative velocity of the satellite in absolute and differential terms [8]. B. Saedeleer (2006) The analytical theory of the Moon's third body, was studied using the Lie transform method for averaging Hamiltonians in cases of synchronous rotation, the Lunar oblateness, the Lunar triaxiality, and the significant influence of the Earth's lunisolar rotation (ELP 2000) [9]. According to Beutler (2006), the paper discusses the development of effective methods for predicting the orbits of low-earth-orbiting objects (LEOs) in the presence of imprecisely characterized force fields. Pseudo-stochastic pulses, piecewise constant accelerations, or piecewise linear and continuous accelerations are used to compensate for the force field's deficiencies [10]. Xua Guangyan, Luo Jianfu and others (2014) Using the Reference Satellite Variable, this paper derives equations for low earth satellites and their relative motion under lunar perturbation (RSV). The derivation incorporates some plausible assumptions to simplify the results and emphasize the third body's influence [11]. Chihabi Yazan and Ulrich Steve (2021) Using conventional orbital components, this article presents an analytical solution for the relative velocity of two spacecraft. The analytical solution is obtained by forward propagating the orbital elements in time and accounting for gravitational field perturbations up to the fifth harmonic, third-body, and drag secular and periodic perturbations and calculating the relative motion in the local-vertical–local-horizontal reference frame at each time step. Compared to a numerical simulator, the analytical solution accurately characterized the relative motion, with errors on the order of meters at separation distances of hundreds of meters [12]. Elisa Maria Alessi, Alberto Buzzoni and others (2021). The purpose of this study is to evaluate the orbital development of the mean eccentricity as defined by the Molniya satellites constellation's Two-Line Elements (TLE) set. The bottom-up technique is used to achieve synergy between observable dynamics and mathematical simulation. With the long-term development of eccentricity as the primary emphasis, the dynamical model used is a doubly-averaged formulation of the third-body disturbance caused by the Sun and Moon and the oblateness influence on the satellite's orientation. The findings demonstrate that the second-order expansion captures the behavior remarkably well despite the very elliptical orbits. Additionally, the lunisolar influence is not negligible for the behavior of the ascending node's longitude and the pericenter's argument. Finally, a frequency series analysis is suggested in order to demonstrate [13]. Due Yujun and Zhang Fang Zhao (2021): Theoretically and numerically, we explore these effects using Gaussian equations of motion. According to the study, PNPM's effect may be classified into two groups. The first component is a rotational error in the perturbing force vector caused by the force vector being converted to a coordinate system without accounting for PNPM effects; the second component is an error in the satellite coordinates computed in the Earth-Centered Earth-Fixed (ECEF) or True-of-Date (TOD) coordinate system without accounting for PNPM effects. Additionally, a straightforward semi-analytical correction strategy is shown. Keplerian elements can be employed instantly without requiring a recalculation of the solutions. This method effectively corrects the error produced when the PNPM was disregarded [14].

Using the celestial mechanics software program version one with assistance of program MATLAB (2018) The aim of this work is to analyze the orbital evolution of the mean orbital elements given by the Two-Line Elements (TLE) set of the (Sun-synchronous) satellite constellation and examine the third body perturbation of the orbit of satellite for various objectives and missions. Demonstrating to how it affects orbital elements as well as coordinates components, and acceleration components at different values of Nodes.

2. Sun-synchronous orbit

Sun-synchronous orbits in which the ascending node's secular rate of right ascension is equal to the mean sun's right ascension rate. To be sun-synchronous, the inclination, semi-major axis, and eccentricity of the satellite orbit. A typical sun-synchronous orbit has an inclination of 98.7 degrees and a mean orbit height of 833 kilometers. Circular orbits at low height [15].

$$\left(\frac{d\Omega}{dt}\right)_s = -\frac{3}{2} n\, J_2 \left(\frac{R}{P}\right)^2 \cos i = 0.9856\ \frac{deg}{day}$$

Were

$n = \sqrt{\frac{\mu}{a^3}}$ orbit mean motion

R = Earth equatorial radius

$p = a(1 - e^2)$

3. Simulation of satellite motion in Sun-synchronous orbit via numerical models

The motion of Sun-synchronous near-circular orbit with input data given by the Two-Line Elements (TLE) set of mean altitude of $7569\, km$ and inclination of $i = 100.6^{\circ}$ so on $e = 0$ and Argument of perigee equal to 0 with values of Nodes ($\Omega = 0, 80, 120, 280, 360$) and period of time 101.2 minutes [16][17]. local equatorial crossing time at 12:00 hours. Significant perturbing factors, such as the Moon's and Sun's pull, were considered. The perturbing bodies' coordinates were obtained using numerical ephemerides DE200/LE200) as constant variables. Using the Celestial Mechanics software system's SATORB module (Beutler, 2005) [18]. Since the integration is performed over a long period of time and no connection to individual data is required, the beginning epoch of January 1, 2000, was chosen for ease of use of the numerical model. The integration was done using the 12th order collocation approach with integration order of 12 and automatic step selection 6000 seconds in 180 days of age with tabular interval of 1 minutes. The model used in this process which is Joint Gravity model (JGM3) the JGM model was developed with NASA and the university of Texas in 1994 [19][18].

4. Perturbation due to third body

The equations of motion of two-point masses M and m when they interact are as follows

$$M\ddot{\vec{r}}_M = GMm\frac{\vec{r}_{Mm}}{r_{Mm}^3} \quad and \quad m\ddot{\vec{r}}_m = GMm\frac{\vec{r}_{mM}}{r_{mM}^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Where r is the vector's length, index Mm indicates that the vector points from point-mass M to point-mass m and single index M or m indicates that the vector points to point-mass M or m. By introducing additional point masses m(j), j = 1, 2,..., the attraction of m(j) on M and m may be expressed, and summations can calculate the total attraction [20].

$$M\ddot{\vec{r}}_M = GM_m\frac{\vec{r}_{Mm}}{r_{mM}^3} + \sum_j GMm(j)\frac{\vec{r}_{Mm(j)}}{r_{Mm(j)}^3}$$

$$m\ddot{\vec{r}}_m = GMm\frac{\vec{r}_{mM}}{r_{mM}^3} + \sum_j Gmm(j)\frac{\vec{r}_{mm(j)}}{r_{mm(j)}^3}$$

By dividing the two preceding equations by $-M$ and m and then adding them together, one obtains

$$\ddot{\vec{r}}_m - \ddot{\vec{r}}_M = -G(M+m)\frac{\vec{r}_{Mm}}{r_{mM}^3} + \sum_j Gm(j)\left\{\frac{\vec{r}_{mm(j)}}{r_{mM}^3} - \frac{\vec{r}_{Mm(j)}}{r_{Mm(j)}^3}\right\} \dots\dots\dots\dots \quad (2)$$

Letting $\vec{r} = \vec{r}_m - \vec{r}_M$ using the point mass (M) as the origin substituting $\vec{r}_{mm(j)} = -(\vec{r}_m - r_{m(j)})$ in the right side of equation (2) and removing the mass m (mass of satellite)

$$\ddot{\vec{r}} = -GM\frac{\vec{r}}{r^3} - \sum_j Gm(j)\left\{\frac{\vec{r} - \vec{r}_{m(j)}}{\left|\vec{r} - \vec{r}_{m(j)}\right|^3} + \frac{r_{m(j)}}{r_{m(j)}^3}\right\} \dots\dots\dots\dots\dots\dots\dots\dots.. (3)$$

It is self-evident that the first component on the right represents the earth's core force; hence, the disturbance forces of various point masses acting on the satellite are then calculated.

$$\vec{f}_m = -m\sum_j Gm(j)\left\{\frac{\vec{r} - \vec{r}_{m(j)}}{\left|\vec{r} - \vec{r}_{m(j)}\right|^3} + \frac{\vec{r}_m}{r_{m(j)}^3}\right\} \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (4)$$

Where Gm(j) denotes the sun, moon, and planets' gravitational constants [20].

5. Collocation method

The collocation approach is used to solve the problem of starting value. Collocation algorithms use a polynomial of degree q to estimate the initial value issue inside the subintervals I_k, which is (in general) greater than Euler's approach. (The collocation algorithm is simplified to the Euler algorithm for q = n.) The order of the technique is also known as the polynomial degree $q \geq n$. The problem with the interval as the starting value may be stated as:

$$y_k^{(n)} = f(t, y_k, \dot{y}_k, \dots, y_k^{(n-1)})$$

$$y_k^{(i)}(t_k) = y_{k0}^{(i)} \quad i = 0,1, \dots., n-1$$

Where $y_{k0}^{(i)}$ is initial value

The collocation algorithm of order $q \geq n$ uses a polynomial of degree q to approximate the initial value issue or the boundary value problem in the interval I_k= $[I_k, t_k + 1]$.

$$y_k(t) = \sum_l^q \frac{1}{l!}(t - t_k)^l \; y_{k0}^{(l)}$$

Within the interval I_k, the differential equation system was solved by numerical solution at exactly $q + 1 - n$ distinct epochs t_{k_j} , $j = 1,2, \dots$, $q + 1 - n$

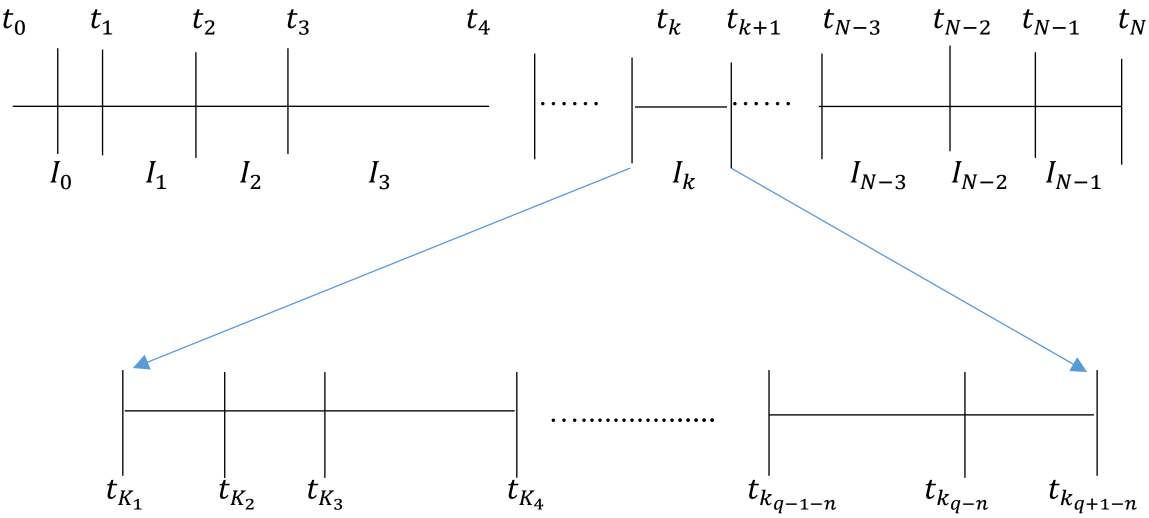


Figure (1): For the collocation algorithm, the integration interval $\boldsymbol{I_k}$ is divided [18].

6. Variation of parameters

The majority of analytical and numerical solutions are based on Euler's and Lagrange's variation of parameters (VOP) versions of the equations of motion. Because the orbital components (the constant parameters in the two-body equations) are changing, the whole process is called a variation of parameters (VOP). Both Lagrange and Gauss devised VOP techniques for analyzing perturbations—approach Lagrange's applies to conservative accelerations, but Gauss's method is also applicable to non-conservative accelerations. The shape will vary depending on the orbital components used; we can study the impact of perturbations using the VOP approach. This is extremely valuable for planning and analyzing missions. Any theory should be able to model as many perturbing forces as feasible. Most operational, analytical theories focus exclusively on the central body and drag. Third-body and solar-radiation forces have significantly fewer analytical formulas because their impacts are substantially less over many orbits. Additionally, while precision demands the utilization of third-body effects and solar radiation pressure, numerical integration is typically just as simple for all perturbing variables [21].

6.1 Conservative forces Lagrangian VOP

The VOP technique is well-suited for generating the equations of motion of perturbed dynamical systems. The concept is founded on the assumption that if the solution's constants are extended to be time-varying parameters, we may utilize the unperturbed system to represent the solution to the perturbed system. The unperturbed system is a two-body system that consists of a series of formulae for determining the position and velocity vectors at a given time. Bear in mind that these computations depend on the six orbital components and time. We may theoretically employ any set of unchanged motion constants, including the original position and velocity vectors. Time is related to motion equations via mean, eccentric, and actual anomaly conversions. The Lagrangian planetary equations of motion, or simply the Lagrangian VOP, are the essential theory for calculating the orbital components' rates of change. It is named after Lagrange since he is credited with formulating these equations for the first time for all six orbital elements. He was mesmerized by the minute perturbations of planets' orbits around the Sun caused by their gravitational attraction [21].

The planetary equation of Lagrange for the orbit of a celestial body in a two-body situation is stated as [18].

$$\dot{a} = \mp \frac{2}{n^2 a} \frac{\partial R}{\partial \dot{T}_0} \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots . (5)$$

$$\dot{e} = -\frac{\sqrt{|1-e^2|}}{n\, a^2 e} \frac{\partial R}{\partial \omega} - \frac{1-e^2}{n^2 a^2 e} \frac{\partial R}{\partial \dot{T}_0} \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (6)$$

$$\frac{di}{dt} = -\frac{1}{n\, a^2 \sqrt{|1-e^2|} \sin i} \frac{\partial R}{\partial \Omega} + \frac{\cot i}{n\, a^2 \sqrt{|1-e^2|}} \frac{\partial R}{\partial \dot{T}_0} \dots \dots \dots \dots \dots . . (7)$$

$$\dot{\Omega} = \frac{1}{n a^2 \sqrt{|1-e^2|} \sin i} \frac{\partial R}{\partial i} \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (8)$$

$$\dot{\omega} = \frac{\sqrt{|1-e^2|}}{n\,a^2 e}\frac{\partial R}{\partial e} - \frac{\cot i}{n\,a^2\sqrt{|1-e^2|}}\frac{\partial R}{\partial i} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

$$\dot{T}_0 = \frac{2}{n^2 a}\frac{\partial R}{\partial a} + \frac{1-e^2}{n^2 a^2 a}\frac{\partial R}{\partial e} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

6.2 Non-conservative forces Gaussian form

It is sometimes more convenient to represent disturbing accelerations directly at the satellite in componential form rather than using partial derivatives of the disturbing potential in the elements. This is especially true for orbits with a large eccentricity, for which series expansions would need a large number of terms in e. Gauss proposed a feasible alternative form. Three mutually perpendicular components are used to resolve the perturbing forces operating on the satellite [21].

As an example, consider the following collection of Gaussian perturbation equations to reduce (RSW) [18].

$$\dot{a} = \sqrt{\frac{p}{\mu}}\frac{2a}{1-e^2}\left\{e\sin vR + \frac{p}{r}S\right\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$\dot{e} = \sqrt{\frac{p}{\mu}}\ \{\sin v\,R + (\cos v + \cos E)\,S\,\} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12)$$

$$\frac{di}{dt} = \frac{r\cos u}{n\,a^2}\,W \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$$\dot{\Omega} = \frac{r\sin u}{n\,a^2\sqrt{1-e^2}\ \sin i}\,W \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

$$\dot{\omega} = \frac{1}{e}\sqrt{\frac{p}{\mu}}\left\{-\cos v\,R + \left(1+\frac{r}{p}\right)\sin v\,S\right\} - \cos i\,\dot{\Omega} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

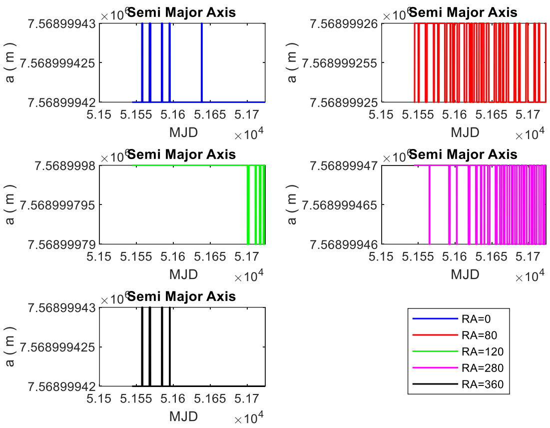
$$\dot{T}_0 = -\frac{1-e^2}{n^2\,ae}\left\{\left(\cos v - 2e\frac{r}{p}\right)R - \left(1+\frac{r}{p}\right)\sin v\,S\right\} - \frac{3}{2a}\left(t - \dot{T}_o\right)\dot{a} \ldots (16)$$

Where v is true anomaly, E is eccentric anomaly and $u = \omega + v$ is the argument of latitude of the celestial body. The perturbation equations above are divided into two groups, the first of which contains the equations for the semimajor axis a (which defines the size), the eccentricity e (which defines the shape), and the time T0 of pericenter passage (which defines the dynamics) of the orbital motion, and the second of which contains the three Eulerian angles I and, which define the orbital plane and the orientation of the conic section within it [22].

7. Application and results

The results demonstrate the influence of third body perturbation of Sun Synchronous orbit selected for long period of time the data obtained from software program celestial mechanics version one which solves the equations of motion analytically using collocation method based on numerical results from the program in terms of two output files one of orbital elements calculated after solving the equations of perturbation programmatically and other one contains the geocentric coordinates axis and acceleration components also MATLAB (2018) was used to analyze and plot the data were the given by celestial mechanics program

(a)



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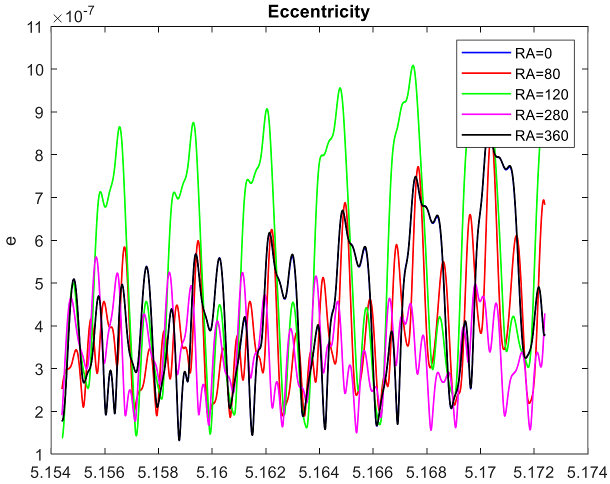

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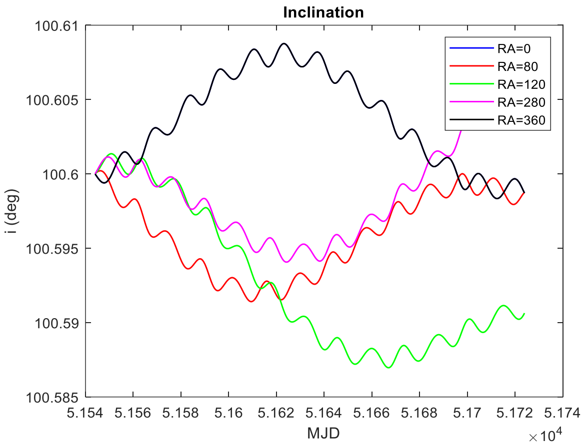

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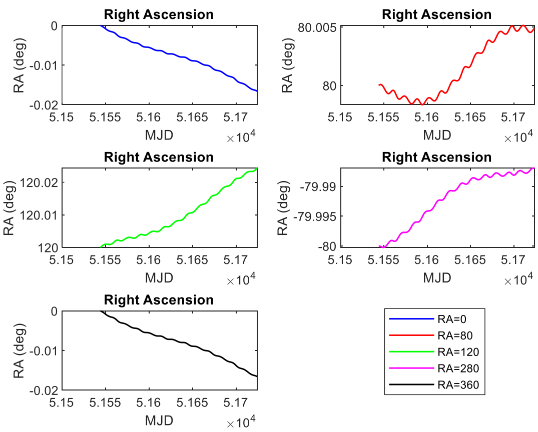

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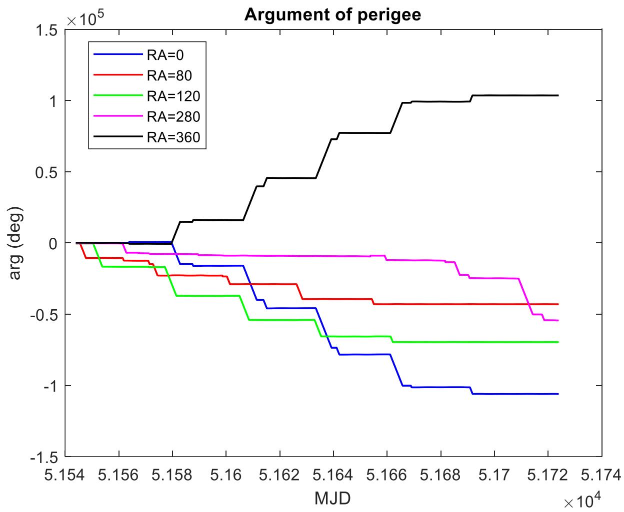

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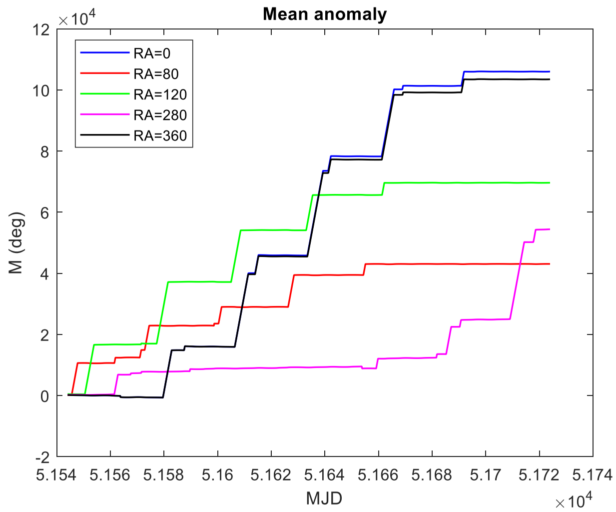
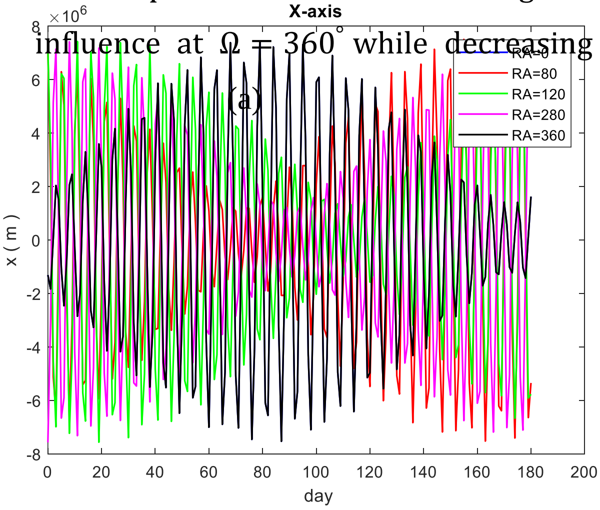
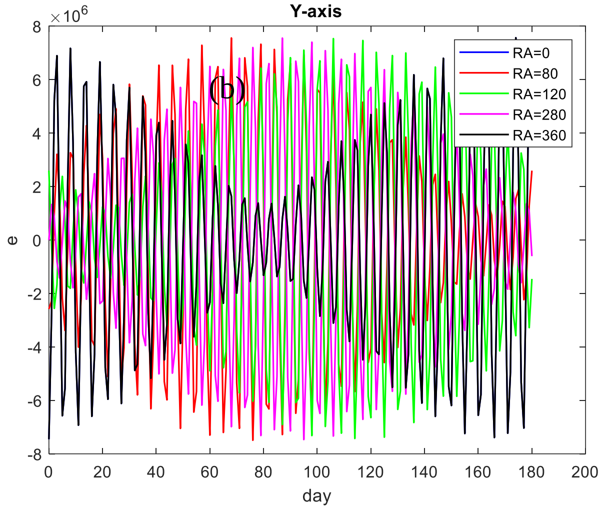


Figure (2): Evolution of Third body perturbation on Orbital elements of Sun-synchronous satellite. Where (a) denotes the Semi Major axis, (b) denotes eccentricity, (c) denotes inclination, (d) denotes right ascension, (e) denotes perigee argument, and (f) denotes mean anomaly.

The effect of the perturbation on the six orbital elements is illustrated in Figure (2), with the element of semi-major axis (a) given in units of (m) has short periodic plus secular influence with greatest effect at $\Omega = 80^{\circ}$, the eccentricity element also exhibiting short and secular effect with increasing rate of change of perturbation, in the inclination secular change are considered for perturbation influence with increasing rate for perturbation effect with highest influence at $\Omega = 360^{\circ}$ while decreasing rate for $\Omega = 180^{\circ}$, in right ascension the rate of change is appears opposite as in inclination which have increasing secular effect in Sun perturbation while decreasing influence is in Moon attraction, as well as the Argument of perigee and Mean anomaly, exhibiting a noticeable lack of influence in terms of secular perturbation The data indicate that the moon's influence is growing compared to the sun, where the third body has a negligible effect in low orbits.



(a)



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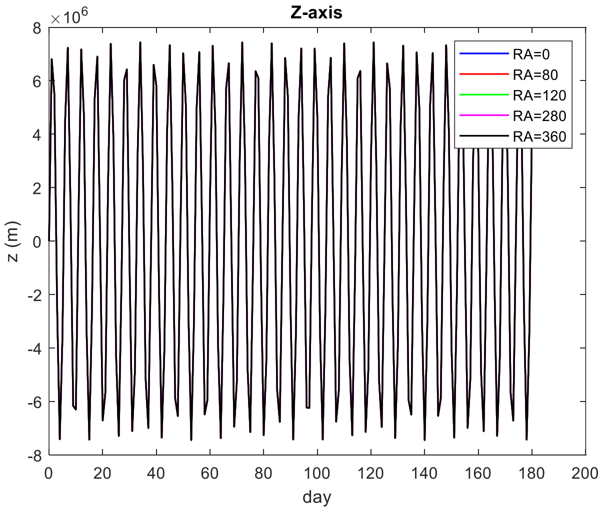
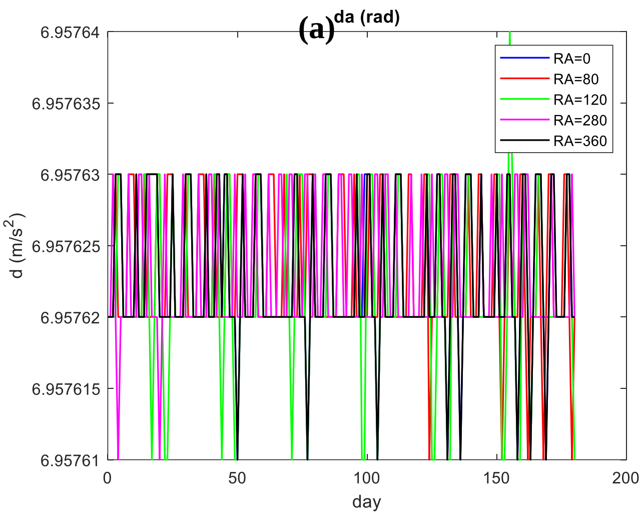


Figure (3): Evolution of Third body perturbation on Geocentric equatorial coordinates system of Sun-synchronous satellite. Where: (a) represent X-axis, (b) Y-axis, (c) Z-axis

Figure (3) shows that the study includes a section of perturbation in geocentric equatorial coordinates (X, Y and Z) given in units of meters. The final results reveal that whereas perturbations of the third body have a similar effect on low–altitude satellites but a bit different in some values of Nodes with short periodic influence in the x-axis, which has units of meters with time given in (days), as shown in the figure, the rate of change is at highest effect at point of $\approx 8 \times 10^6\ m$ for x-axis and for y-axis and z-axis, they have a more significant influence on coordinates system as periodic effect and the net effect of the third body perturbation is on three-axis given.

(a)



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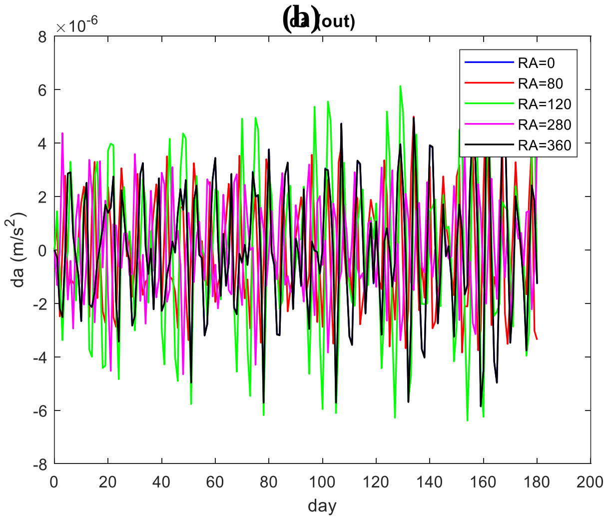

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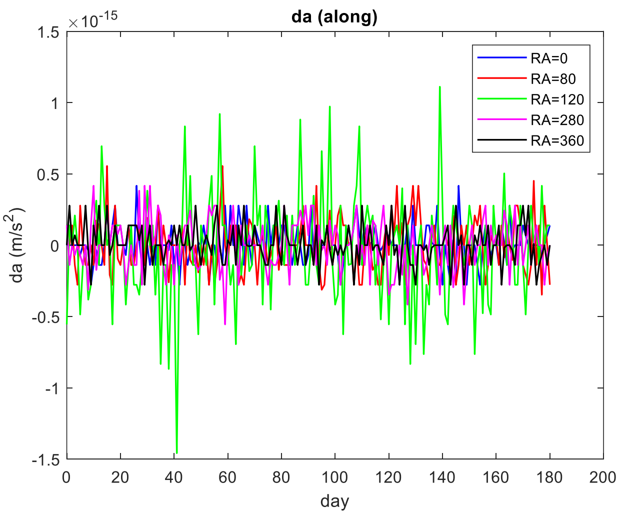

Figure (4): Evolution of Third body perturbation on Acceleration components system of Sun-synchronous satellite Where: (a) represent radial, (b) along-track, (c) out of plane.

The perturbation influence on acceleration components (radial, along-track, and out-of-plane) directions, as seen in figure (4). The third body has a little influence significant on the radial term with secular plus short periodic effect at the point $\approx 6.95763\ m/s^2$ with the non-stable rate of change. For out of plane direction, the effect has periodic influence with greatest effect at rate of $6 \times 10^{-6}\ m/s^2$ and minimum rate effect at $-6 \times 10^{-6}\ m/s^2$ for $\Omega = 120^{\circ}$. In the side of a long-track motions, the perturbation influence is shortly periodic plus secular and has the greatest rate of change with increasing rate at point of $1 \times 10^{-15}\ m/s^2$ for $\Omega = 120^{\circ}$.

8. Conclusion

The influence of the third body perturbation on satellites at Sun-synchronous altitudes was researched in this work for many values of Right Ascension of node ($\Omega = 0$, $\Omega = 80$, $\Omega = 120$, $\Omega = 280$, $\Omega = 360$). It was discovered that the value of the perturbation of the third body increased and decreased with orbital elements as well as with geocentric axis or acceleration components each of force effect apply in a different type of perturbation as periodic and secular. The data obtained a slight influence of the third body on the acceleration components of satellite furthermore greatest effect was in Argument of perigee at $(1 \times 10^{6})\ deg$ for $\Omega = 360^{\circ}$ and at $(11 \times 10^{4})\ deg$ for $\Omega = 0°, 360^{\circ}$ of Mean anomaly in orbital elements and have highest effect at $\approx (8 \times 10^{6})\ m$, which owns due to orbits altitude compared with force type when the air drag should have more influence on this satellite the study aims to show the small data effects on the satellite for these orbits with different area . Finally, Due to perturbations force, the set of orbital elements and Geocentric coordinates axis are influenced mainly by the force of perturbation as appeared periodic and secular from numerical results obtained from the program

REFERENCES

[1] david A. Vallado, *FUNDAMENTALS OF ASTRODYNAMICS AND APPLICATIONS*, Fourth Edi. Published jointly by Microcosm Press and Springer, 2013.

[2] L. Rogres, *It's only rocket Science*. © 2008 Springer Science+Business Media, LLC, 2008.

[3] W. Torge and J. Müller, *Geodesy*, 4th Editio. © 2012 Walter de Gruyter GmbH & Co. KG, Berlin/Boston.

[4] G. X. • J. Xu, *Orbits*, Second Edi. # Springer-Verlag Berlin Heidelberg 2013, 2013.

[5] Y. Kozai, "The motion of a close earth satellite," *Astron. J.*, vol. 64, p. 367, 1959, doi: 10.1086/107957.

[6] Yoshihide kozai, "A NEW METHOD TO COMPUTE LUNISOLAR PERTURBATIONS IN SATELLITE MOTIONS," *Smithson. Astrophys. Obs.*, 1973.

[7] Martin Lara · Juan F. San-Juan · Luis M. López and P. J. Cefola, "On the third-body perturbations of high-altitude orbits," *Celest Mech Dyn Astr*, 2012.

[8] · C. W. T. R. · S. R. V. and K. T. Alfriend2, "Third-Body Perturbation Effects on Satellite Formations," *J Astronaut Sci*, 2015.

[9] B. De Saedeleer, "Analytical theory of a lunar artificial satellite with third body perturbations," *Celest. Mech. Dyn. Astron.*, 2006.

[10] G. B. · A. J. · U. H. · L. Mervart, "Efficient satellite orbit modelling using pseudo-stochastic parameters," *J. Geod.*, 2006, doi: 10.1007/s00190-006-0072-6.

[11] Guangyan Xua, J. Luo, Z. Li, and X. Chen, "Equations of Satellite Relative Motion in Low Earth Orbit under Lunar Perturbation," *Proc. 2014 IEEE Chinese Guid. Navig. Control Conf.*, 2014.

[12] S. U. Yazan Chihabi ∗, "Analytical spacecraft relative dynamics with gravitational, drag and third-body perturbations," *Acta Astronaut.*, 2021, doi: 10.1016/j.actaastro.2021.04.043.

[13] E. M. Alessi, A. Buzzoni, J. Daquin, A. Carbognani, and G. Tommei, "Dynamical properties of the Molniya satellite constellation: Long-term evolution of orbital eccentricity," *Acta Astronaut.*, pp. 659–669, 2021, doi: 10.1016/j.actaastro.2020.11.047.

[14] Y. Du, Fangzhao Zhang, T. Xu, F. Gao, and G. Xu, "Correction of precession-nutation and polar motion in analytical solutions of satellite equations of motion," *Adv. Sp. Res.*, 2021, doi: 10.1016/j.asr.2021.07.041.

[15] V. A. Chobotov, *Orbital Mechanics*, Third Edit. American Institute of Aeronautics and Astronautics, Inc., 2002.

[16] N2yo, "n2yo." https://www.n2yo.com/.

[17] CelesTrak, "NORAD Two-Line Element Sets." https://www.celestrak.com/NORAD/elements/.

[18] G. Beutler, *Methods of Celestial Mechanics Volume II:* Springer-Verlag Berlin Heidelberg 2005, 2005.

[19] M. Capderou, *Satellites Orbits and Missions*. © Springer-Verlag France 2005, 2005.

[20] G. X. • Y. Xu, *GPS*, Third Edit. © Springer-Verlag Berlin Heidelberg, 2016.

[21] P. Gurfil, *Modern Astrodynamics*, First edit. 2006, Elsevier Ltd. All rights reserved, 2006.

[22] G. Beutler, *Methods of Celestial Mechanics Volume I:* © Springer-Verlag Berlin Heidelberg 2005, 2005.