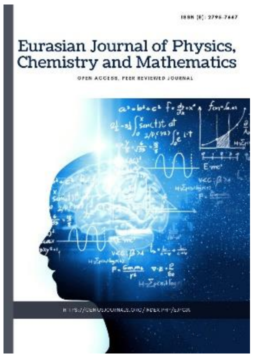

A boundary problem for a single class of third-order equations

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ABSTRACT

The boundary problem for a third-order equation of mixed type in a four-angle region is considered. Using the Galerkin method, under certain conditions, the existence of a weakly generalized solution in Sobolev's space was proved on the coefficients and the right side of the equation. Under the same conditions, the uniqueness of the generalized solution is proved

Keywords: Galerkin method, boundary problem, third-order equation

The theory of boundary value problems for equations of mixed type, due to its applied and theoretical significance, has become one of the most important sections of the theory of partial differential equations. The beginning of the study of boundary value problems for equations of mixed type was laid in the works of F. Tricomy and S. Gellerstedt, where boundary value problems for model equations of mixed type were first posed and studied. The bibliography of the question can be found in the monograph [1], [2]. Later in the works [3], [4], the proposed substitutions were generalized for high-order equations, for various boundary conditions for operator equations and for equations of a non-classical type. Among the latest works can be noted [5], [6].

The L_p, W_p^l. definitions and properties of these spaces can be found in the monograph [7].

In the field $Q=\{(x,t):-1\le x\le 1,\ 0\le t\le T\}$, consider

$$Lu\equiv u_{ttt}+\mu(x)u_{xxx}+a(x,t)u_{xx}+b(x,t)u_{tt}+c(x,t)u_x+d(x,t)u_t+e(x,t)u=f(x,t) \quad (1)$$

where $x\mu(x)>0$ at $x\neq 0$, $\mu(0)=0$.

Boundary problem. Find in the field Q a solution of equation (1) satisfying the boundary conditions

$$u\big|_{\partial Q}=0,\ \ u_t\big|_{t=0}=0. \quad (2)$$

For the prostate, we assume that the coefficients of equation (1) are infinitely differentiable functions.

Definition 1. Denote through the $H(Q)$ space of functions obtained by closing functions from $C^{\infty}(Q)$, satisfying conditions (2) according to the norm

$$\|u\|_{H(Q)}=\int_Q (u_{xx}^2+u_{xt}^2+u_x^2+u_t^2+u^2)dQ$$

Definition 2. The function $u\in H(Q)$ will be called a weak generalized solution of the problem (1), (2), if the $v\in C_0^{\infty}(Q)$ identity is fulfilled for all

$$\int_Q (u_t v_{tt} - \mu u_{xx} v_x - \mu_x u_{xx} v + a u_{xx} v - b u_t v_t - b_t u_t v + c u_x v + d u_t v + e u v) dQ = \int_Q f v dQ \quad (3)$$

Thoseaboutthe rema. Let the conditions be met

$$a(x,t) - \frac{3}{2}|\mu_x| \geq \delta > 0, \quad b(x,t) \geq 0,$$

(4)

then for any function $f(x,t)$ such that $f \in L_2(Q)$ there exists a single solution to the problem (1), (2) of $H(Q)$.

Proof. Solution of problem (1), (2) we will look for the Galerkin method

$$u_m(x,t) = \sum_{i=1}^{m} j_i(t)\varphi_i(x)$$

where $\varphi_i(x)$ functions are solutions to the problem

$$\varphi_i'' = -\lambda_i \varphi_i, \quad \varphi_i(-1) = \varphi_i(1) = 0.$$

And the coefficients $j_i(t)$ are from the solution of the system of ordinary differential equations

$$(u_{mttt}, \varphi_i)_0 + (\mu u_{mxxx}, \varphi_i)_0 + (a u_{mxx}, \varphi_i)_0 + (b u_{mtt}, \varphi_i)_0 + (c u_{mx}, \varphi_i)_0 + (d u_{mt}, \varphi_i)_0 +$$
$$+(e u_m, \varphi_i)_0 = (f, \varphi_i)_0 \quad (5)$$

$$j_i(0) = j_i(T) = j_{it}(0) = 0, \quad i = 1, 2, \ldots, m \quad (6)$$

The solvability of the problem (5), (6) with a fixed m one follows from the general theory of ordinary differential equations.

Let's get uniform m estimates for the Galerkin approximations. To do this, multiply (5) by $-j_i(t)$ and, summing up by i, the result.

$$(u_{mttt}, -u_m)_0 + (\mu u_{mxxx}, -u_m)_0 + (a u_{mxx}, -u_m)_0 + (b u_{mtt}, -u_m)_0 + (c u_{mx}, -u_m)_0 +$$
$$+(d u_{mt}, -u_m)_0 + (e u_m, -u_m)_0 = (f, -u_m) \quad (7)$$

From here, integrating the po t, and integrating parts into force (2), (4), after some transformations we come to inequality

$$\int_Q (u_{mt}^2 + u_{mx}^2 + u_m^2) dQ \leq C \quad (8)$$

Next, consider the following equality

$$(u_{mttt}, u_{mxx})_0 + (\mu u_{mxxx}, u_{mxx})_0 + (a u_{mxx}, u_{mxx})_0 + (b u_{mtt}, u_{mxx})_0 =$$
$$= (f - c u_{mx} - d u_{mt} - e u_m, u_{mxx})_0 \quad (9)$$

From identity (9), strength (2), (3) and evaluation (8), integrating by t, parts and integration in parts, after simple transformations follows the following assessment

$$\int_Q (u_{mxx}^2 + u_{mxt}^2) dQ \leq C. \quad (10)$$

From the estimates (8), (10) follows the limitation of the sequence of approximate solutions $\{u_m(x,t)\}$ in space $H(Q)$, you can select subsequences $\{u_{m_k}(x,t)\}$ and go to the limit on $m_k \to \infty$ in the system (5). It is not difficult to verify that the limit function belongs to the space $H(Q)$ and satisfies the identity (3). Because the system $\{\varphi_i(x)\}$ is dense in $L_2(-1,1)$.

Let us prove that the solution of the problem (1), (2) is the only one.

If $u, v-$two solutions to the problem (1), (2), then $w = u - v$ satisfies the equation

$$w_{ttt} + \mu(x) w_{xxx} + a(x,t) w_{xx} + b(x,t) w_{tt} + c(x,t) w_x + d(x,t) w_t + e(x,t) w = 0$$

Consider the integral

$$\int_Q (w_{ttt} + \mu(x) w_{xxx} + a(x,t) w_{xx} + b(x,t) w_{tt} + c(x,t) w_x + d(x,t) w_t + e(x,t) w) w dQ = 0$$

and integrating in parts into force (2) we get

$$\int_Q (w_t^2 + w_x^2 + w^2)dQ \leq 0$$

It follows that $w = 0$ in Q.

Ttheorem is proven.

In this paper, new theorems of uniqueness and existence of a solution for boundary problems (1), (2) are obtained, which make it possible to expand the rangeof problems in the theory of boundary value problems for non-classical equations of mathematical physics.

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