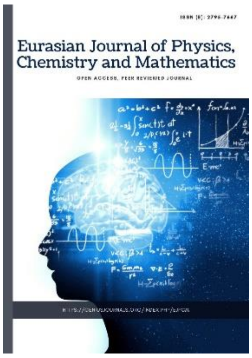

Nonlocal problem for a single fourth-order equation

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ABSTRACT

This paper examines a nonlocal problem for a single fourth-order equation. If some conditions are met, the existence and uniqueness of the solution of the problem are proved on the right side of the equation.

Keywords: Non-local task, conjugate problem, fourth-order equalization.

Nonlocal problemsare now a developing branch of the theory of differential equations. Interest in a nonlocal problem (except for the theoretical value) is obviously caused by the possibility of its physical interpretation: if the differential equation describes some physical process, then nonlocal boundary conditions are described by some algebraic expressions linking the desired solution and its derivatives at two or more points of observation of the physical process. The problems of modern science and technology have put forward the solution of more real practical problems associated with the study of various classes of mathematical models. It is known that theathematic modeling of many biological and technological processes leads to the study of nonlocal boundary value problems for various classes of differential equations. Therefore, the doctrine ofnonlocal problems for different classes of differential equations attracted the attention of many mathematicians. The bibliography of the question can also be foundin monographies [1], [2]. The environment of the latest works can be noted [3] – [7].

In the field, $Q=\{(x,y,t):\ 0\le x\le 1,\ 0\le y\le 1,\ 0\le t\le T\}$ consider the equationof the fourth order

$$Lu\equiv u_{ttxx}-u_{xxxx}-u_{yyxx}+u_{xx}=f(x,y,t) \qquad (1)$$

Non-local task. Find a solution to Q equation (1) in the field that satisfies the following conditions

$$u\big|_{t=0}=e^{\frac{\lambda T}{2}}u\big|_{t=T},\quad u_t\big|_{t=0}=e^{\frac{\lambda T}{2}}u_t\big|_{t=T},\quad u\big|_{x=0}=u\big|_{x=1}=u_x\big|_{x=0}=u_x\big|_{x=1}=0,\ u\big|_{y=0}=u\big|_{y=1}=0 \qquad (2)$$

where $\lambda>0-$ some numbers.

It is not difficult to see that the next task for (1), (2) will be a conjugate task

$$L^{*}v\equiv v_{ttxx}-v_{xxxx}-v_{yyxx}+v_{xx}=g(x,y,t) \qquad (3)$$

$v\big|_{t=0}=e^{-\frac{\lambda T}{2}}v\big|_{t=T}$, $v_t\big|_{t=0}=e^{-\frac{\lambda T}{2}}v_t\big|_{t=T}$, $v\big|_{x=0}=v\big|_{x=1}=v_x\big|_{x=0}=v_x\big|_{x=1}=0,\ v\big|_{y=0}=v\big|_{y=1}=0$ (4)

Denote by C_L (respectively C_{L^*}) a class of functions that are four times continuously differentiated in a domain Q and satisfy condition (2) (respectively (4)).

Definition 1. Denote through $H(Q)$ (respectively $H(Q)^*$) the space of functions obtained by closing functions from C_L (respectively from C_{L^*}) according to the norm

$$\|u\|^2=\int_Q(u_{ttx}^2+u_{txx}^2+u_{txy}^2+u_{tt}^2+u_{tx}^2+u_{xx}^2+u_{xy}^2+u_t^2+u_x^2+u_y^2+u^2)dQ.$$

Definition 2. The function $u\in H(Q)$ will be called a weak generalized solution of the problem (1), (2), if the $v\in C_{L^*}$ identity is fulfilled for all

$$B(u,v)=\int_Q(u_{xtt}v_x+u_{xx}v_{xx}+u_{xy}v_{xy}+u_{xx}v)dQ=\int_Q fvdQ,\ \forall v\in C_{L^*}.$$

Theorem. Let the condition for the function $f(x,y,t)\in W_2^1(Q)$ be satisfied

$$f\big|_{t=0}=e^{\frac{\lambda T}{2}}f\big|_{t=T},\quad \int_0^x f(w,y,0)dw=e^{\frac{\lambda T}{2}}\int_0^x f(w,y,T)dw \qquad (5)$$

There is a single solution to the problem (1), (2) of $H(Q)$.

Proof. Multiplying equation (1) multiplying by $e^{\lambda t}u_t$ and integrating over the domain Q, and by virtue of condition (2), (3) we get

$$\int_Q(u_{tx}^2+u_{xx}^2+u_{xy}^2+u_x^2)dQ\le C\int_Q f^2dQ. \qquad (6)$$

Equations (1) can be rewritten as

$$u_{ttx}=u_{xxx}+u_{xyy}+u_x+\int_0^x f(w,y,t)dw+\varphi(y,t) \qquad (7)$$

where $\varphi(x,y)-$is the unknown function. From (7), in force of (3),n

$u_{xtt}\big|_{x=0}=u_{xxx}\big|_{x=0}+\varphi(y,t)=0$ Ie $\varphi(y,t)=-u_{xxx}\big|_{x=0}$

since

$\varphi(y,0)=-u_{xxx}(0,y,0)$ and $\varphi(y,T)=-u_{xxx}(0,y,T)$.

From here, in force (2), we get

$$\varphi(y,0)=e^{\frac{\lambda T}{2}}\varphi(y,T).$$

Thenyes from equation (7) we derive that

$$u_{xtt}\big|_{t=0}=e^{\frac{\lambda T}{2}}u_{xtt}\big|_{t=T} \qquad (8)$$

Further, equation (1) differentiating by t, then multiplying by $e^{\lambda t}u_{tt}$ and integrating by the region Q, due to conditions (2), (6), (8), pirradiate

$$\int_Q(u_{ttx}^2+u_{txx}^2+u_{txy}^2+u_{tx}^2)dQ\le C\int_Q(f^2+f_t^2)dQ \qquad (9)$$

Since, by virtue of (2), (3), for the solution of task (1), (2) the estimate is correct

$$\int_Q(u_{ttx}^2+u_{txx}^2+u_{txy}^2+u_{tt}^2+u_{tx}^2+u_{xx}^2+u_{xy}^2+u_t^2+u_x^2+u_y^2+u^2)dQ\le C\int_Q(f^2+f_t^2)dQ \quad (10)$$

Now consider the conjugate problem (3), (4). Similarly, as in (5),multiplying equations (3) by $-e^{-\lambda t}v_t$ and integrating over the region Q and in force (4), (8) we get

$$\int_Q (v_{tx}^2 + v_{xx}^2 + v_{xy}^2 + v_t^2)dQ \le C\int_Q g^2 dQ. \qquad (11)$$

Note that as in (8), ve rnoequality

$$v_{xtt}\big|_{t=0} = e^{-\frac{\lambda T}{2}} v_{xtt}\big|_{t=T} \qquad (12)$$

Further, equation (3) differentiating by t, then multiplying by $-e^{-\lambda t}v_{tt}$ and integrating by the region Q, integrating parts by force (4),(11),(12) we get

$$\int_Q (v_{ttx}^2 + v_{txx}^2 + v_{txy}^2 + v_{tt}^2)dQ \le C\int_Q g_t^2 dQ \qquad (13)$$

From the estimates (10), (11), (13) it follows in a standard way the existence and uniqueness of the solution of the problem (1), (2) [8 – 10] (see, for example, [9, p. 19, p. 35]) in the space $H(Q)$. The theorem is proven.

It is not difficult to see that problem (1), (2) can be interpreted as an inverse problem for determining the right side of the wave equation. This note proves new theorems of existence and uniqueness of solving a nonlocal problem (1), (2), which make it possible to expand the range of solvable problems in the theory of nonlocal problems for non-classical equations of mathematical physics.

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