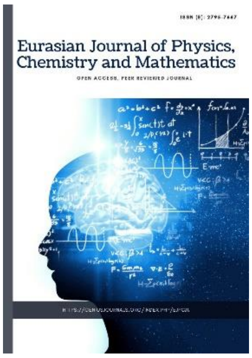

The Numerical Solution of Gas Filtration in Hydrodynamic Interconnected Two-Layer Reservoirs

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ABSTRACT

This article discusses specific problems of gas filtration in two-layer reservoirs. For the solution, the method of lines with a combination of simple factorization is applied. For the numerical solution of the problem, the Maple software system was used.

Keywords: Gas filtration, two-layer reservoir, method of lines, permeability, filtration, convergence, accuracy, time step.

We will study the numerical solution of gas filtration when a two-layer reservoir consists of a well-permeable layer bounded from below (from above) by weakly permeable interlayers. We are investigating transient flow to perfectly vertical wells drilled in a highly permeable layer. Under some assumptions, the non-stationary boundary value problem in dimensionless form is formulated as follows, i.e. is described by the system of equations

$$\begin{cases} \dfrac{1}{m(x)}\dfrac{\partial}{\partial x}\left(k(x)\dfrac{\partial u}{\partial x}\right) = M(u)\dfrac{\partial u}{\partial t} + A(x)k(z)\left.\dfrac{\partial u_1}{\partial z}\right|_{z=1} \\ \dfrac{1}{m_1(x)}\dfrac{\partial}{\partial z}\left(k_1(z)\dfrac{\partial u_1}{\partial z}\right) = M_1(u_1)\dfrac{\partial u_1}{\partial t},\ x,z,t \in Q_T \end{cases} \quad (1)$$

at initial

$$u(x,0)=\varphi(x),\ u_1(x,z,0)=\varphi_1(x,z),\ (x,z)\in\Omega \quad (2)$$

and boundary conditions

$$k(x)\left.\frac{\partial u}{\partial x}\right|_{x=0} = q_0(t),\ k(x)\left.\frac{\partial u}{\partial x}\right|_{x=1} = q_{n+1}(t),$$

$$k_1(z)\left.\frac{\partial u_1}{\partial z}\right|_{z=0} = 0,\ u(x,t)=u_1(x,1,t),\ (x,z,t)\in Q_T \quad (3)$$

Pairing (matching) condition

$$\left.[u]\right|_{x=x_k}=0,\ \left.\left[k(x)\frac{\partial u}{\partial x}\right]\right|_{x=x_k} = q_k(t),\ k=\overline{1,n} \quad (4)$$

Where

$$Q_T=\Omega\times(0,1)\times(0,T],\ \Omega=\bigcup_{k=0}^{n}\Omega_k,\ \Omega_k=(x_k,x_{k+1})$$

Assume that, with the smoothness of the coefficients of the system of equations, there is a unique solution to the initial-boundary value problems (1)-(4), [1].

For an approximate solution (1)-(4). Let us cover with straight lines $t = t_j$, where

$$t_j = j\tau,\ j = 1, 2, \ldots, N;\ N = \left[\frac{T}{\tau}\right]$$

Denote by $\{u_j(x),\ u_{ij}(x, \mathrm{z})\}$ desired approximate value of the desired function $\{u(x,t),\ u_1(x,\mathrm{z},t)\}$ on a straight line $t = t_j$. We approximate problems (1)-(4) by the following scheme

$$\begin{cases} \dfrac{1}{m(x)}\dfrac{d}{\partial x}\left(k(x)\dfrac{du_j}{dx}\right) = M\left(u_{j-1}\right)\delta_{\bar{t}} u_j + A(x)k_i(z)\dfrac{\partial u_1}{\partial z}\Big|_{z=1} \\ \dfrac{1}{m_1(z)}\dfrac{\partial}{\partial z}\left(k_1(\mathrm{z})\dfrac{\partial u_{ij}}{\partial z}\right) = M_1\left(u_{ij-1}\right)\delta_{\bar{t}}\, \mathrm{u}_{ij} \end{cases}$$

(5)

With the following conditions

$$u_0(x) = \varphi(x),\ u_1(x, \mathrm{z}) = \varphi_1(x, z),$$

$$k(x)\frac{du_j}{dx}\Big|_{x=0} = q_0\left(t_j\right),\ k(x)\frac{du}{dx}\Big|_{x=1} = q_{n+1}\left(t_j\right),$$

$$\left[u_j\right]\Big|_{x=x_k} = 0,\quad k(x)\frac{du_j}{dx}\Big|_{x=x_k} = q_k\left(t_j\right),$$

$$k_1(z)\frac{\partial u_1}{\partial z}\Big|_{z=0} = 0,\quad u_j(x) = u_{1j}(x, \mathrm{z}),$$

Let the coefficients of the system of equations (5) be calculated $t = t_{j-1}$. Then each of equations (5) is linear with respect to $\{u_j,\ u_{ij}\}$ and has a unique solution [5-11].

For a numerical solution, we will change the function

$$V_j = u_j + F_{1j}(x) - F_{2j}(x)$$

$$V_{1j} = u_{1j} + F_{1j}(x) - F_{2j}(x)$$

Where

$$F_{1j}(x) = q_0\left(t_j\right)\sigma(x) - q\left(t_j\right)R(x)$$

$$F_{1j}(x) = \begin{cases} 0 \ \ npu \quad x \in \overline{\Omega} \\ \displaystyle\sum_{i=1}^{k} q_i\left(t_j\right)\int_{x_i}^{x}\frac{d\lambda}{k(\lambda)} \quad npu\ \ x \in \overline{\Omega} \end{cases}$$

$$q\left(t_j\right) = q_{n+1}\left(t_j\right) - \sum_{k=1}^{n} q_k\left(t_j\right)$$

$$\sigma_0(x) = \int_x^1 m(\xi)d\xi, \qquad \sigma_1(x) = \int_0^x m(\xi)d\xi,$$

$$\sigma(x) = \frac{1}{\sigma_0(0)}\int_x^1\frac{\sigma_0(\lambda)}{k(\lambda)}d\lambda, \qquad R(x) = \frac{1}{\sigma_1(1)}\int_0^x\frac{\sigma_1(\lambda)}{k(\lambda)}d\lambda$$

Then we arrive at the following boundary value problem for systems of ordinary differential equations

$$\begin{cases} \dfrac{1}{m(x)}\dfrac{d}{dx}\left(k(x)\dfrac{dV_j}{\partial x}\right) = \dfrac{M\left(u_{j-1}\right)}{\tau}V_j - \dfrac{M\left(u_{j-1}\right)}{\tau}\left[F_{1j}(x) - F_{2j}(x)\right] + \\ +\left[\dfrac{q_0\left(t_j\right)}{\sigma_0(0)} - \dfrac{q\left(t_j\right)}{\sigma_1(1)}\right] + A(x)k_1(z)\dfrac{dV_{1j}}{\partial z}\Big|_{z=1} - \dfrac{M\left(u_{j-1}\right)}{\tau}V_{j-1} \\ \dfrac{1}{m_1(z)}\dfrac{\partial}{\partial z}\left(k_1(\mathrm{z})\dfrac{\partial u_{ij}}{\partial z}\right) = \dfrac{M\left(u_{ij-1}\right)}{\tau}V_{ij} - \dfrac{M_1\left(u_{j-1}\right)}{\tau}\left[F_{1j}(x) - F_{2j}(x)\right] - \dfrac{M_1\left(u_{ij-1}\right)}{\tau}V_{ij-1} \end{cases}$$

$$\frac{du_j}{dx}\Big|_{x=0} = \frac{dV_j}{dx}\Big|_{x=1} = 0,\quad \left[V_j\right]_{x=x_k} = 0, \left[k(x)\frac{dV_j}{dx}\right]\Big|_{x=x_k} = 0,\quad k_1(z)\frac{\partial V_{ij}}{\partial x}\Big|_{z=0} = 0,\ \mathrm{V}_{1j}(x,1) = V_j(x)$$

The approximate solution constructed by the method of lines converges at a rate $0(\tau)$ [2].

For numerical implementation, we will use the method of a modified version of the sweep differential. Consider a model boundary value problem.

$$\begin{cases} \dfrac{\partial}{\partial x}\left(k(x)\dfrac{\partial u^2}{\partial x}\right) = 2m\mu\dfrac{\partial u}{\partial t} + Ak_1(z)\dfrac{\partial {u_1}^2}{\partial z}\Big|_{z=1} \\ \dfrac{\partial}{\partial z}\left(k_1(z)\dfrac{\partial {u_1}^2}{\partial z}\right) = 2m_1\mu\dfrac{\partial u_1}{\partial t} \end{cases}$$

$$\{0 < x < 1,\ 0 < z < H_1,\ 0 < t \le T\}$$

With initial conditions

$$u(x,0) = u_1(x, \mathrm{z}, 0) = u_0 = cons`t,\ 0 < x < 0,\ 0 < z \le H_1$$

And boundary conditions

$$b_0 H_1\frac{k(x)}{\mu}\frac{u}{u_{0m}}\frac{\partial u}{\partial x}\Big|_{x=0} = Q(t)$$

$$k(x)\frac{\partial u}{\partial x}\Big|_{x=1} = 0$$

$$u_1(x,1,t) = u(x,t)$$

$$k_1(z)\frac{\partial u_1}{\partial z}\Big|_{z=0} = 0$$

Here - u_0 initial formation pressure distribution

$k_1(z)$ and $k(x)$ reservoir permeability

$\mu_1,\ \mu_2$ - the thickness of the layers, $m,\ m_1$ - the formation porosity

Q(t) - daily consumption.

We pass to dimensionless variables according to the formula

$$\xi=\frac{x}{L};\ \eta=\frac{z}{H_1};\ P_1=\frac{u_1}{u_0};\ P=\frac{u}{u_0}$$

$$k_1^*(\eta)=\frac{k_1(z)}{k_{xop}},\ \ k(\xi)=\frac{k(x)}{k_{xop}}$$

$$t^*=\frac{k_{xop}u_0}{2m\mu L^2}t$$

Where $A^4=\frac{L^2}{H_1A_2}$, $M=\frac{m_1}{m}\frac{A_1^2}{L}$ u_0- initial plastic pressure distribution

$K_2(x)$ u $K_1(z)-$ permeability of the first and second formation, u_1 u u_2 $-$ layer thickness, θ_o^*- daily consumption of the gallery, b_0- seam width, $K_{xap}-$ and characteristic permeability of the formation.

$$\begin{cases}\dfrac{\partial P}{\partial t^*}=\dfrac{\partial}{\partial\xi}\left(K^*(\xi)\dfrac{\partial P^2}{\partial\xi}\right)-A^*K_1^*(\eta)\dfrac{\partial P^*}{\partial\eta}\bigg|_{\eta=1}\\ M\dfrac{\partial P_1}{\partial t^*}=\dfrac{\partial}{\partial\eta}\left(K_1^*(\eta)\dfrac{\partial P_1^2}{\partial\eta}\right)\end{cases}$$

$$P_1(\xi,\eta,0)=P_2(\xi,0)=u_0=const$$

$$K^*(\xi)\frac{\partial P_2^2}{\partial\xi}\bigg|_{\xi=0}=Q^*(t),\quad K^*(\xi)\frac{\partial P^2}{\partial\xi}\bigg|_{\xi=1}=0$$

$$P_1(\xi,1,t)=P(\xi,t),\quad K_1(\eta)\frac{\partial P_1^2}{\partial\eta}\bigg|_{\eta=0}=0$$

To illustrate, consider an example with the following data:

$$u_0=150ai;\quad m_0=0.01;\ m_1=0.2;\ \ \mu=0.01cn\textit{з};\ K_0(x)=4\partial apcu;\ K_1(z)=0,01\sqrt{\frac{z}{H_1}}\partial apcu$$

$$Q=10^5m^3/cyt,;\quad b_0=50m;\ H_1=200m;\ H_2=20m$$

Table 1 compares the calculation of the problem $P_1(\xi,\eta,t^*)$ u $P(\xi,t^*)$ with different steps τ at $t^*=3.24$. Table 2-3 gives an approximate value $P_1(\xi_1,\eta,t^*)$ and $P(\xi_1,t^*)$ calculated in time steps $\delta=6.62$ and $\tau=3{,}9$ for a different point in time t^* at $t^*=0.48$.

The accuracy of the numerical values of the pressure field is controlled by the balance equation, in which for a given problem, the deviation of the approximate values from the exact ones does not exceed at $t^*=3.24$; 0.1% throughout the calculation interval.

The calculation is based on the application of the Maple system (to solve differential equations), we use the Runge-Kutta method

Table 1. The Runge-Kutta method

ξ \ η	0	0.5	1	
1	0.9889	0.9883	0.9889	1.62
	0.9889	0.9883	0.9899	3.24
0.5	0.9892	0.986	0.9892	1.62
	0.9893	0.9886	0.9893	3.24
0	0.9894	0.9887	0.9894	1.62
	0.9894	0.9888	0.9894	3.24

z^*	3.24			32.4			64.8		
ξ \ η	0	0.5	1	0	0.5	1	0	0.5	1
1	0.9889	0.9883	0.9889	0.8886	0.8878	0.8886	0.7770	0.7762	0.7770
0.5	0.9882	0.0886	0.9892	0.8889	0.8882	0.8889	0.7774	0.7766	0.7774
0	0.9894	0.9887	0.9894	0.8891	0.8884	0.8891	0.7776	0.7768	0.7776
P_{cp}	0.989820			0.989938			0.780245		
$\int_0^1 P_2(\xi,t)d\xi$	0.988671			0.888305			0.776721		
$q_1(t^*)$	0.44306 10-1			0.44654			0.89382		
$K_n(t^*)$	0.6645			0.6690			0.6702		

$$K_n(t^*) = -\frac{q_1(t^*)}{\int_0^t Q^* d\tau}$$

z^*	3.24			32.4			64.8		
$\eta \backslash \xi$	0	0.5	1	0	0.5	1	0	0.5	1
1	0.9889	0.9883	0.9889	0.8883	0.8885	0.8893	0.7785	0.7777	0.7785
0.5	0.9893	0.9886	0.9893	0.8896	0.8889	0.8896	0.7789	0.7783	0.7789
0	0.9894	0.9888	0.9894	0.8898	0.8893	0.8898	0.7791	0.7789	0.7791
P_{cp}	0.988761			0.889276			0.778815		
$\int_0^1 P_2(\xi,t)d\xi$	0.988739			0.889004			0.778221		
$q_1(t^*)$	0.4408 10^{-1}			0.44421			0.89096		
$K_n(t^*)$	0.6628			0.6678			0.6682		

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