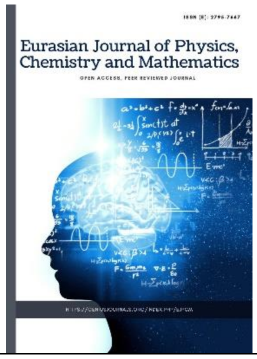


Graphs and Their Applications in Various Fields

N.S. Kurbonova
Master of Tashkent State Pedagogical University named after Nizami

ABSTRACT
The article provides information on the theory of graphs, types of graphs and the application of graphs to various fields. A number of problems have been solved using graphs as examples. The article also contains a number of recommendations for math teachers on the use of graphs.

Keywords: Graph Theory, Graph, Graph Tip, Graph Edge.

In 1736, L. Euler proposed and solved the problem of the Kyonisberg bridges, which was an interesting and practical problem of the time, which gave rise to the theory of graphs. In the middle of the nineteenth century, research on the theory of graphs appeared in the works of G. Kirchhoff and A. Keli. The term "graph" appears in the first textbook on graph theory by D. Kyonig in 1936.

We will now turn to the abstract definition of a graph and the basic concepts associated with a graph. Let V be some empty set. We denote by $V \times V$ (the set itself a Cartesian multiplication) the set of all pairs (tuples) of the form $< v_1, v_2 >$, consisting of its elements $v_1 \in V$ and $v_2 \in V$. A graph is a pair $< V, U >$ where $V \neq \emptyset$ and U is a cortege of pairs of $< v_1, v_2 >$ $(v_1 \in V, v_2 \in V)$ form, composed of elements of the set $V \times V$. Given a graph $G = (V, U)$. The elements of set V are called the ends of the graph G, and the set V itself is called the set of graph ends. Graph theory sometimes uses the word "node" or "dot" instead of "three." In general, no general consensus has yet been reached on some terms of graph theory. So we try to use more alternative sentences.

According to the definition of the graph $G = (V, U)$, U can also be an empty cortege. If U is not empty, then this procession consists of pairs of the form (a, b) $(a \in V, b \in V)$, where $a = b$ and an arbitrary pair $a = b$ U can participate in the procession as desired. $(a, b) \in U$ it can be called different depending on the order of the ends a and b, which make up the pair, ie the presence or absence of direction. If the order of the components for the pair (a, b) is insignificant, that is, $(a, b) = (b, a)$, then (a, b) is a non-oriented edge or, in short, an edge is called. If this order is important, i.e. $(a, b) \neq (b, a)$, then the pair (a, b) is called an arc or an oriented edge. U depending on the composition of the cortege, we call it either a cortege of the edges of a graph, or a cortege of arcs, or a cortege of edges and arcs. $G = (V, U)$ number of graph elements $|V| + |U|$ where G is the number of vertices of the graph $|V| \neq 0$ and $|U|$ is determined by the number of its edges (arcs). The edges (arcs) of a graph are usually defined by the ends that make it up (a, b) or ab. Other definitions are also used: for example, for the arc $\overleftarrow{(a,b)}$ or $\overrightarrow{(b,a)}$, for the edge $\overline{(a,b)}$, for the arc or edge u (i.e. the ends co by means of a single letter). It is important to note that for an arc of a graph, the order in which its endpoints are shown is important, i.e., arcs (a,b) and (b,a) represent arcs that differ from each other. If the arc (a,b) is expressed in the form, then a is called its beginning and b is called its end. In addition, if the arc is written in the form (a,b), then it is customary to call it an arc that starts from a edge (beginning) and b enters from edge (ending at edge). For an edge (a,b), the position of the letters in the text does not matter, and the elements a and b are called the ends or edges of the edges. If a graph contains (a,b) an arc, or (a,b) an arc, or (b,a) an arc, then the ends a and b are said to be connected. If there is an edge or arc connecting the two ends of the graphs, then they are called adjacent ends, otherwise they are called non-adjacent ends. If the two ends of a graph are adjacent, the incident to the edge (arc) that connects those three ends is called the incident, and the edge or arc to these three is called the incident. When two edges (arcs) have a common edge in a graph, they are called adjacent edges (arcs). It should be noted that the concept of neighborhood represents the homogeneity of the graph, and the concept of incident represents the relationship between its different gender elements. Sometimes a graph is defined by the number of elements in it, ie the number of ends m and the number of edges (arcs) n, in which case the graph (m,n) is called a graph. If in a graph $G=(V,U)$ the U cortege consists only of oriented edges (ie arcs), then this graph is called a oriented graph. An oriented graph is also called a short graph. In some cases, you have to work with graphs that have both oriented edges and non-oriented edges. Such graphs are called mixed graphs. If the U cortege of a graph (orgraf) $G=(V,U)$ contains repeating elements from the set $V\times V$, then they are called multiple or parallel edges (arcs). A graph with multiple edges or arcs is called a multigraph. The edge (arc) whose two (beginning and end) edges overlap, that is, the element of the graph $(a,a)\in U$, is called the halter. Halter is usually considered non-directional. A graph with halters between its edges is called a pseudograph. Edge isolated (separated, neutral, bare) edge that are not connected by any arc are called alone edge. A graph consisting only of isolated ends (that is, the graph has no edges and no ends) is called a zero graph or an empty graph. It is customary to denote an empty graph with the number of vertices m as O_m or N_m. A non-oriented graph with no adjacent ends and multiple edges is called a complete graph. The complete graph with the number of ends m is denoted by K_m. Obviously, the number of edges of the graph K_m is $C_m^2=\frac{m(m-1)}{2}$. If there is only one arc connecting each of the two ends of the orggraf in each direction, then it is called a full orggraf. Obviously, if you replace each of the edges in a full graph with two (opposite directions) arcs, the result is a complete graph. Therefore, the number of arcs in a full graph is twice the number of edges in an unoriented full graph, that is, the number of arcs in a full graph with ends m is $2C_m^2=m(m-1)$. If the end of a graph has any symbols, such as the numbers 1, 2, 3,..., m, it is called a definite graph.

If the sets of ends of the graphs $G=(V,U)$ and $G'=(V',U')$, that is, between the sets V and V', are mutually exclusive if a value match can be established, then graphs G and G' are called isomorphic graphs. This definition can also be expressed as follows: if for $\forall x,y\in V$ and its corresponding $\forall x',y'\in V'$ $(x\leftrightarrow y, x'\leftrightarrow y')$ If $xy\leftrightarrow x'y'$ $(\forall x\, y\in U;\ \forall x',y'\in U')$ then the graphs G and G' are isomorphic. If one of the isomorphic graphs is oriented, then the other must also be oriented, and the directions of the corresponding arcs in them must also be aligned. The number of incident edges on the end of a graph is called the local level, or abbreviated level, the valence of that end. The degree of a edge in the graph is denoted by $\rho(a)$.

In determining the extent of the three incidents to the slip, it should be borne in mind that, depending on the subject under consideration, the slip can be considered as one-sided or two-sided. Obviously, the degree of the separated three is zero. Three edges (or hanging) equal in level together are called three. The incident edge of the edge (hanging) three is also called the edge (hanging) edge.

If all the ends of a graph have the same degree r, then such a graph is called a regular graph of degree r. A three-level regular graph is called a cubic (or three-valent) graph. Note that O_m is a zero-degree regular graph and K_m is a $(m-1)$ regular graph. It can be seen that in an oriented graph, the sum of all the levels of the ends is an even number equal to twice the number of edges, because when counting the edges, each edge is involved twice in the calculation. Thus, the following assertion proved by L. Euler in the eighteenth century is appropriate:

Lemma (about "meetings"). In an arbitrary non-oriented graph, the sum of all the levels of the ends is twice the number of edges.

If the set of vertices of a graph can be divided into two such non-intersecting sets (segments), then the arbitrary edge of the graph connects the arbitrary triangle from one of these sets with any three from the second set. If so, then such a graph is called a two-segment graph (bichromatic or cyanotic graph). It is clear from the definition that any two ends of a two-segment graph cannot be adjacent. A complete two-part graph with only one end is called a star. If any two or three adjacent parts of a two-part graph belong to different parts, then such a graph is called a complete two-part graph. We denote a complete two-part graph by $K_{m,n}$, where m and n denote the number of ends in the parts of the graph. For the graph $K_{m,n} = (V, U)$ it is clear that $|V| = m + n$ and $|U| = mn$, where $|V|$ - $K_{m,n}$ is the number of ends of the graph, $|U|$ the number of edges of the graph.

We will now give examples of graphs.

For example. We denote the set of airports in the territory of the Republic of Uzbekistan by V, and the procession of flights between cities by U for a given period of time. In that case (V, U) the pair can be considered as a graph. Here, the ends of the graph correspond to the airports, and the arcs correspond to the landings of planes. Of course, the graph (V, U) can also have multiple arcs, and if for some reason the plane lands back at the airport where it flew, then the graph in the graph under consideration is appropriate.

For example. Let's look at the following issue, which is one of the oldest puzzles. Divide the 8-volume liquid in a container into two equal parts using the same container and 5 and 3-volume containers. Determining the volumes of liquids in 8, 5 and 3 unit volume vessels with a, b and c, respectively, we construct the ends $< a, b, c >$ representing the state of the system under consideration based on the volumes of liquid in the vessels for a given time. According to the condition of the problem, the variables a, b, c must satisfy the conditions $0 \leq a \leq 8$, $0 \leq b \leq 5$ and $0 \leq c \leq 3$, assuming all values. The circumstances that satisfy these conditions are: $< 8,0,0 > < 7,1,0 > < 7,0,1 > < 6,2,0 > < 6,1,1 > < 6,0,2 > < 5,3,0 > < 5,2,1 > < 5,1,2 > < 5,0,3 > < 4,4,0 > < 4,3,1 > < 4,2,2 > < 4,1,3 > < 3,5,0 > < 3,4,1 > < 3,3,2 > < 3,2,3 > < 2,5,1 > < 2,4,2 > < 2,3,3 > < 1,5,2 > < 1,4,3 > < 0,5,3 >$.

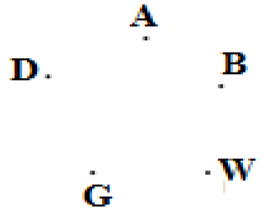
We denote the set of states by V. Pouring a liquid (or part of it) from one container to another can result in a change from one state to another. It should be noted that it may not be possible to switch directly or indirectly to any of the above cases. We denote by U it the set of possibilities of the system's transition from one state to another. The resulting pair (V, U) can be considered as a graph. The ends of this graph correspond to the states of the system, and the arcs (edges) correspond to the direct transitions. To solve this problem, we need to create a sequence consisting of the arcs of the graph (V, U) such that the first term of this sequence is $< 8,0,0 >$, and the last term is $< 4,4,0 >$. Here is one such sequence:

$< 8,0,0 >$, $< 5,0,3 >$ $< 5,3,0 >$ $< 2,3,3 >$ $< 2,5,1 >$ $< 7,0,1 >$ $< 4,1,3 >$ $< 4,4,0 >$.

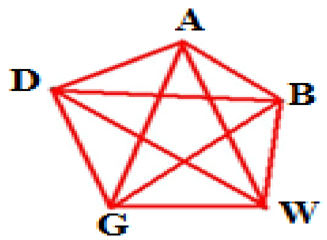
Here are some examples of applications of graphs in different fields.

For example. Akbar, Botir, Wahob, Ghayrat and Dilmurod met. They shook hands when they saw each other (each shaking hands with the other once). What is the total number of handshakes?

Solution. We place 5 points on the plane. We assume that each of them is one of the people named in the case. To avoid mistakes, we capitalize the names of these people.



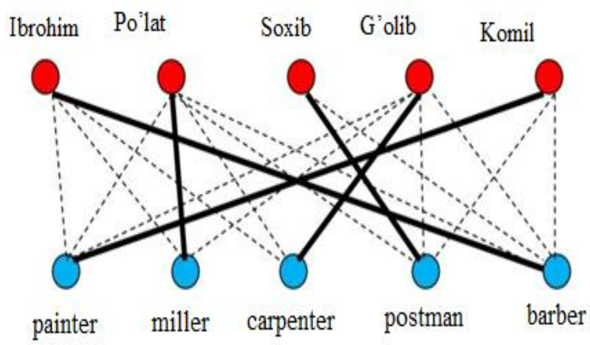
Now let's connect these points with lines (i.e. edges). If we count the number of edges in this graph, that's exactly the number of handshakes.



Answer: The number of handshakes is 10.

For example. Friends Ibrohim, Po'lat, Soxib, G'olib and Komil live in a small town. They have different professions: one is a painter, another is a miller, another is a carpenter, and the last is a postman. Steel and Winner have never held a paintbrush in their hands. Ibrohim and G'olib want to go together to see their friends who work at the mill. Po'lat and Komil live in the same house with their postman friends. Steel and the barber's daughter got married together. Soxib attended the wedding as a guest. Ibrohim and Po'lat play with the painter and carpenter every weekend. G'olib and Komil meet every Saturday at the barber shop where their friends work. The postman prefers not to use the services of a barber. Find the occupation of each of the friends named above.

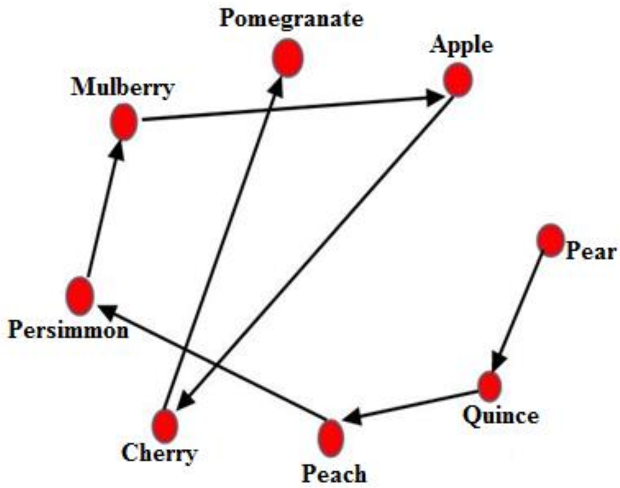
Solution.



Answer: So Ibrohim is a barber, Po'lat is a miller, Soxib is a postman, G'olib is a carpenter, and Komil is a painter.

For example. In the village, 8 fruit trees grow in Nadir's father's yard. They are pomegranate, apple, pear, quince, peach, cherry, persimmon and mulberry. Higher than a date peach. Quince is taller than a pear. Apples are lower than cherries, but higher than mulberries. Mulberry is taller than a date. Cherries are lower than pomegranates. Peaches are taller than quinces. What is the order of these trees in descending order from the longest of their height?

Solution. We mark the trees as the ends of the graph. Then, according to the condition of the problem, we draw an arc from a short tree to a relatively long tree (i.e., an edge with a direction) so that the arrow pointing in the direction is on the side of the taller tree.



As you can see from the graph, the trees are arranged in the following order: pomegranate, cherry, apple, mulberry, persimmon, peach, quince and pear.

In the early days of graph theory, graphs were considered simply interesting mathematical problems, but today they have become a major part of mathematics. Significantly, the theory of graphs has practical significance not only in the field of mathematics, but also in various other fields. Interest in graph theory, in particular, has grown significantly in recent decades. The scope of graph theory is growing every year. For example, the theory of graphs is also used in the field of "logistics", which in recent years has become a very developed field. In this area, intercity, interstate products are transported.

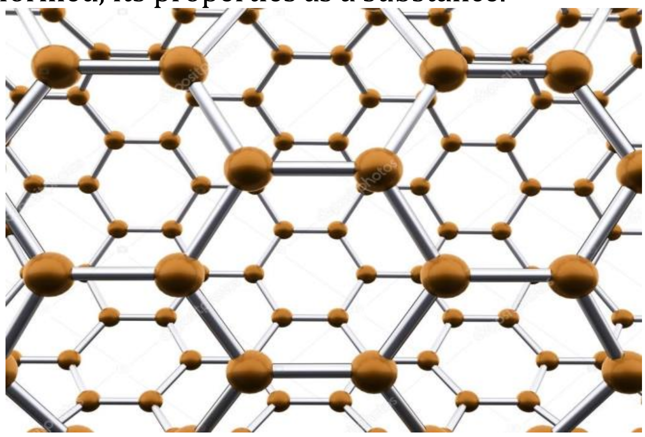
The person in charge of this process, as a logistics master, must organize this process in the most convenient way (ie, the cheapest, the least fuel-efficient, the safest). In this case, he must have a lot of knowledge, but also be aware of the theory of graphs. Well, the question is, how can graph theory help in this? In this case, the cities where the cargo is to be picked up and left are assumed to be the ends of the counts.

We know that the edges of graphs can be connected with different edges. These two ends are connected in different ways (that is, in different ways). Then the most optimal (most convenient) of these edges is selected. This leads to an increase in productivity. There are also many types of graphs and areas of application. For example, graphs are used in chemistry, computer engineering, tourism, and computer programming. Here are some of them:

Application of graph theory in chemistry

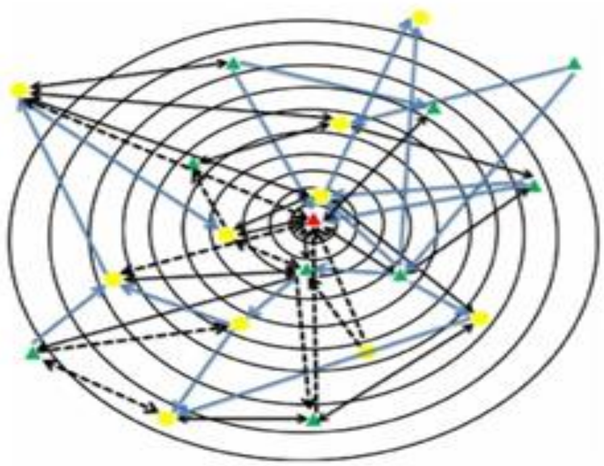
Graph theory is used in chemistry to solve a variety of practical and theoretical problems. Graphs are often used to solve problems related to the structure of substances. For example, each element has a valence. It is on the basis of this valence that the elements combine to form different substances.

Graphs are used to study the properties, composition and structure of the formed substances. In this case, the ends of the graphs are the atoms of the element, and the edges represent their valences. It is these edges that represent the physical state of the substance formed, its properties as a substance.



Application of graph theory in tourism

Each of us, when we go to a new country or city, first of all wants to visit all the places that attract the attention of tourists. But unfortunately time is limited. There is very little time, especially on a business trip. That's why it's important to plan your destinations in advance and choose the most convenient, least time-consuming, and most cost-effective route. This is where graph theory comes in handy. It is a good idea to choose the most convenient route by marking the destinations as the tips of the graphs and the directions as the edges. In addition, buses, subways, ie public transport, all operate on the basis of the count's theory. All stations are the ends of the graph, and the paths between the stations are the edges of the graph.



Graph theory and the internet

Today, the internet has taken over the world. The purpose of the Internet is to store and transmit data. Even the internet works on the basis of graph theory. In this case, we take the Internet sites as the ends of the graph, and the links that go from site to site as edges. There are billions of sites on the Internet, and millions of sites are created and millions of sites are deleted every day. It is from this site that the speed and quality of the transition to the site determines whether the Internet is good or bad.

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