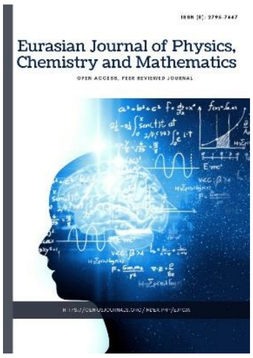

Derivation of Schrodinger Equation from Maxwell Electric Equation

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ABSTRACT

Maxwell's equations can be used to derive Schrodinger equation. The polarization term is used to find the energy term. One can use plank quantum hypothesis beside Einstein mass energy relation to find a useful expression for Maxwell electric equation. Replacing the electric field intensity by the wave function one found Schrodinger quantum equation.

Keywords: Maxwell's equations, Schrodinger equation, plank hypothesis Einstein mass energy relation.

1. Introduction:

The real history of mechanics stars and dates back to the formulation of Newton's laws of mechanics, these laws defines clearly the notions of force, mass, momentum and energy. These laws successfully describe. The behavior of macroscopic objects including astronomical objects [1],[2],[3]. Electromagnetic waves which are classified as useful energy forms, can be easily described by Maxwell's equations. This include light waves [4]. According to these laws the physical word is decided into two categories, particles and matter which are described by Newton's laws and energy which is in the form of waves [5],[6],[7].

According to this version the black body radiation can be described by the wave nature of light. Unfortunately this version fails to describe the black body radiation. This encourages Max plank to propose that light and electromagnetic waves behave. As particles called quanta. The energy of each quanta is proportional to its frequency. This particle nature of electromagnetic radiation and light succeeded in explaining photoelectric effect, pair production, atomic spectra, Compton effect beside black body radiation. This dual (particle – wave) nature of electromagnetic waves was generalized by De Brogglie, who suggested that atomic particles like electrons can behave also as wave. This hypothesis was verified experimentally by the electron diffraction experiment [8],[9]. The dual nature of the atomic world was successfully described by the wave packet notion which was used by Schrodinger together with the Newton energy momentum relation to formulate Schrodinger quantum equation [10],[11]. This equation successfully describes a wide variety of physical phenomena including atomic spectra, Zeeman effect, electron and nuclear magnetic resonance [12]. This success of Schrodinger equation motivates many researchers to use alternative ways to derive it or to promote it this work is concerned with using Maxwell's equation's to derive Schrodinger equation. This is done in section 2. Section (3) and (4) are devotee for discussion and conclusion.

2. Schrodinger Equation From Maxwell's Equation:

Maxwell's equation's describe the behavior of electromagnetic field propagating in a certain medium having electric permittivity ε, magnetic permeability μ and conductivity σ. The electric intensity E satisfies

$$\nabla^2 E - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} - \mu_0 \sigma \frac{\partial E}{\partial t} = 0$$

(1)

Where

$$\mu = \mu_0 \qquad \varepsilon = \varepsilon_0$$

(2)

For vacuum

The conductivity is related to the current density J, electric field intensity and electric flux density according to the relation

$$J = \sigma E = \frac{\partial D}{\partial t}$$

(3)

Where the electric flux density is related to E and ε according to the relation.

$$D = \varepsilon E$$

(4)

One can solve equation (1) by suggesting the solution

$$E = E(r) e^{-iwt}$$

(5)

According to equations (4) and (5), ne gets

$$\frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} = -iwt\ \varepsilon\ E$$

(6)

Using equations (2), (3) and (6) yields

$$J = \sigma E = -iwt \varepsilon_0 w E$$

(7)

Therefore

$$\sigma = -iw\varepsilon_0$$

(8)

The speed of light in free space c satisfies

$$\mu_0\ \varepsilon_0 = \frac{1}{c^2}$$

(9)

Inserting (9) in (2) gives

$$\nabla^2 E - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} - \mu_0 \frac{\partial^2 E}{\partial t^2} - \mu_0 \sigma \frac{\partial E}{\partial t} = 0$$

(10)

In classical physics the speed of light is very large such that

$$c \to \infty$$

(11)

Thus equation (10) becomes

$$\nabla^2 E - \mu_0\ \frac{\partial^2 P}{\partial t^2} - \mu_0 \sigma \frac{\partial E}{\partial t} = 0$$

(12)

For a wave moving in vacuum, the conductivity is given by equation (8). Thus eqn (12) takes the form

$$\nabla^2 E + i\mu_0 \varepsilon_0 w \frac{\partial E}{\partial t} - \mu_0 \frac{\partial^2 P}{\partial t^2} = 0$$

(13)

The electric dipole moment P. Can define the electric susceptibility x in the form

$$P = xE = xE(r)e^{-iwt}$$

(14)

Differentiating w. r. t time gives

$$\frac{\partial P}{\partial t} = -\, iwxE$$

$$\frac{\partial^2 P}{\partial t^2} = i^2 w^2 xE = -w^2 xE$$

(15)

The current density J is related to P according to the relation

$$J = \frac{\partial P}{\partial t} = x \frac{\partial E}{\partial t} = -iwxF = \sigma E$$

(16)

Hence the conductivity is given by

$$\sigma = -iwx$$

(17)

In view of eqns (3) and (4), one gets

$$J = \sigma E = \frac{\partial D}{\partial t} = \varepsilon \frac{\partial E}{\partial t} = -iw\varepsilon F$$

(18)

There fore

$$\sigma = -iw\varepsilon$$

(19)

Combining eqns (16) and (19) yields:

$$\sigma = -iwx = -iw\varepsilon$$

$$x = \varepsilon$$

(20)

Hence

$$\frac{\partial^2 P}{\partial t^2} = -w^2 xE = -w^2 \varepsilon\, E$$

(21)

Inserting eqn (21) in (12) gives

$$\nabla^2 E + \frac{iw}{c^2} \frac{\partial E}{\partial t} + w^2 \mu_0\, \varepsilon\, E = 0$$

(22)

Multiplying both sides by $\left(\frac{-\hbar^2}{2m}\right)$ yields

$$-\frac{-\hbar^2}{2m} \nabla^2 E - \frac{i\hbar^2 w}{2mc^2} \frac{\partial E}{\partial t} - \frac{\hbar^2 w^2}{2m} \mu_0\, \varepsilon\, E = 0$$

(23)

For free space

$$\varepsilon = \varepsilon_0$$

(24)

Therefore

$$\mu_0 \varepsilon = \mu_0 \varepsilon_0 = \frac{1}{c^2}$$
(25)
Inserting eqn (25) in (23) in (23) yields

$$-\frac{-\hbar^2}{2m}\nabla^2 E - \frac{i\hbar^2 w}{2mc^2}\frac{\partial E}{\partial t} - \frac{\hbar^2 w^2}{2mc^2}E = 0$$
(26)

For wave motion the maximum speed $\bar{c}$ is related to the average speed c according to the relation

$$\bar{c} = \sqrt{2}\, C$$
(27)
Therefore the relativistic energy is given by

$$E = m\bar{c}^2 = 2mc^2$$
(28)
Where m is the particle mass.

Now the Maxwell electric equation can be transformed to describe the particle nature by using the plank relation

$$E_n = \hbar w = m\bar{c}^2 = 2mc^2$$
(29)

Using eqn (29) in (26) simplify it to be in the form

$$-\frac{-\hbar^2}{2m}\nabla^2 E \quad i\hbar\frac{\partial E}{\partial t} - \hbar w F = 0$$
(30)
Using eqn (29), yields

$$-\frac{-\hbar^2}{2m}\nabla^2 E - E_n E - i\hbar\frac{\partial E}{\partial t} = 0$$
(31)

The Newtonian energy relation in terms of potential V is given by

$$E_n = \frac{1}{2}mv^2 + V$$
(32)
Where v is the particle speed. For mass less photon

$$m \approx 0$$
(33)

$$E_n = V$$
(34)

But since the photon is emilled from atoms with negative total energy E_n ($E_n = -13.6Z^2/n^2$), it follows that (E is always positive)

$$E_n = -E = -V$$
(35)

Thus using eqn (35) in (31) gives

$$-\frac{-\hbar^2}{2m}\nabla^2 E + VE = i\hbar\frac{\partial E}{\partial t}$$
(36)

Since the electric field intensity is related to the flux density according to eqn (4) and the wave function ψ is related to the particle density. Thus one can easily replace E by ψ in (36) i.e

$$E \to \psi$$
(37)

to get

$$-\frac{-\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

(38)

3. Discussion:

The classical limit of Maxwell's equation requires to get rid of terms which vanishes for large speed of light which approaches infinity according to equation (11). In this case the electric Maxwell's equation reduces to equation (14). The electric susceptibility has been found to be related to the electric permittivity according to equation (20). The conductivity of vacuum has been found to be frequency dependent according to equation (19). All these relations together with the relation of energy with frequency and mass were used to find the final expression for Maxwell electrics equation (36).

Replacing the electric field intensity by the wave function due to their direct proportionality to the intensity I, Schrodinger equation (38). Has been obtained.

4. Conclusion:

Schrodinger equation can be derived from Maxwell's equation's. by considering the speed of light to be infinite in the classical limit and using plank quantum hypothesis beside Einstein mass energy relation a useful expression for Maxwell electric equation was found. Replacing the electric field intensity by the wave function one found Schrodinger quantum equation.

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