A review on integral transforms of the fractional derivatives of Caputo-Fabrizio and Atangana-Baleanu

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ABSTRACT

In this paper, we present a review of the integral transformations of fractional calculus, namely the Riemann-Liouville integral, the Riemann-Liouville derivative, and the Caputo derivative.

Keywords: Integral Transform; Riemann-Liouville integral; Riemann-Liouville derivative; Caputo derivative

1 Introduction

Numerous and varied sectors of engineering and research, including electromagnetics, viscoelasticity, fluid mechanics, electrochemistry, biological population models, optics, and signals processing, use fractional calculus. It has been used to simulate technical and physical processes that fractional differential equations have been determined to best represent. For precise modeling of systems that call for accurate modeling of damping, fractional derivative models are utilized. Numerous analytical and numerical techniques, as well as their applicability to fresh issues, have been put forth in these disciplines recently. The purpose of this special issue on "Fractional Calculus and its Applications in Applied Mathematics and Other Sciences" is to examine the most recent research in the aforementioned areas of fractional calculus conducted by the top scientists. After a thorough and meticulous peer-reviewing procedure, the papers for this special issue were chosen. Fractional differential equations and other issues involving certain mathematical physics functions, as well as their expansions and generalizations in one or more variables, are typically the outcome of mathematical modeling of real-life difficulties. Additionally, fractional order PDEs regulate the majority of physical processes in the models of fluid dynamics, quantum physics, electricity, ecological systems, and many other fields within their valid domain. It is crucial to be familiar with all existing and newly developed techniques for solving fractional order PDEs, as well as the applications of these techniques[1-4].

The integral transform is a mathematical method for translating a differential equation into an algebraic equation. By using this method, a challenging mathematical issue can be reduced to a more manageable one. By integrating the result of a function and another function, known as the kernel of the integral transform, the integral transform mathematical operator can create a result function. The integral transform can be expressed generally as follows:

$$\mathfrak{F}(\mathfrak{s}) = \int \mathfrak{K}(\mathfrak{t},\mathfrak{s})\mathfrak{f}(\mathfrak{s})\mathfrak{d}\mathfrak{s}$$

Where, $\mathfrak{f}(\mathfrak{s})$ represents the function produced by the integral transform and $\mathfrak{K}(\mathfrak{t},\mathfrak{s})$ the kernel function.

A function can be transformed via an integral transform from one domain in which some mathematical operations are quite challenging to another domain in which they are more pliable and easier to comprehend mathematically. Typically, the integral transform's inverse is employed to translate the resultant function back into its original domain [5-7].

Through what was mentioned above, we know the role and importance of fractional calculus and integral transformations, so we will present a review of a number of integral transformations of the Riemann-Lieuvel integral, the Riemann-Liuvel derivative, and the Caputo derivative. The purpose of this is to reduce the effort on researchers and to gain time instead of wasting it in searching for information in multiple sources.

Let $\sigma \in (0,1)$, $\phi \in H^1(a,b)$, $t > a$ and $b > a$, then [8]

- the Atangana-Baleanu-Riemann derivative is given by as

$$ {}^{ABR}_{\ a}\mathfrak{D}^{\sigma}_{t}\phi(t) = \frac{\mathfrak{B}(\sigma)}{1-\sigma}\frac{d}{dt}\int_{a}^{t}\phi(\tau)E_{\sigma}(-\sigma\frac{(t-\tau)^{\sigma}}{1-\sigma})d\tau, \qquad \tau > 0, $$

- the Atangana-Baleanu-Caputo derivative is given by as

$$ {}^{ABC}_{\ a}\mathfrak{D}^{\sigma}_{t}\phi(t) = \frac{\mathfrak{B}(\sigma)}{1-\sigma}\int_{a}^{t}\phi'(\tau)E_{\sigma}(-\sigma\frac{(t-\tau)^{\sigma}}{1-\sigma})d\tau, \qquad \tau > 0. $$

- the Caputo-Fabrizio-Riemann derivative is given by as

$$ {}^{CFR}_{\ a}\mathfrak{D}^{\sigma}_{t}\phi(t) = \frac{\mathfrak{B}(\sigma)}{1-\sigma}\frac{d}{dt}\int_{a}^{t}\phi(\tau)exp(-\sigma\frac{(t-\tau)}{1-\sigma})d\tau, \qquad \tau > 0, $$

- the Caputo-Fabrizio-Caputo derivative is given by as

$$ {}^{CFC}_{\ a}\mathfrak{D}^{\sigma}_{t}\phi(t) = \frac{\mathfrak{B}(\sigma)}{1-\sigma}\int_{a}^{t}\phi'(\tau)exp(-\sigma\frac{(t-\tau)}{1-\sigma})d\tau, \qquad \tau > 0. $$

Where $\mathfrak{B}(\sigma)$ is a normalization function such that $\mathfrak{B}(0) = \mathfrak{B}(1) = 1$ and $E_{\sigma}\left(-\sigma\frac{(t-\tau)^{\sigma}}{1-\sigma}\right)$ is the Mittag-Leffler.

2 Integral Transforms of Fractional Calculus

This section will deal with integrative transformations of the Riemann-Liouville integral and the Riemann-Liouville derivative and the Caputo derivative.

Let $f(t)$ be a continuous function on the interval $[0,\infty)$ which is of exponential order, that is, for some $c \in \mathbb{R}$ and $t > 0$

$$ \sup\frac{|f(t)|}{e^{ct}} < \infty. $$

Now, we know the integral transformations of the integral and the fractional derivative as follows.

A. Laplace transform [9]

$$ \mathcal{L}\{f(t)\} = \int_{0}^{\infty} f(t)e^{-st}\,dt = F(s), \qquad s > 0, $$

1. $\mathcal{L}\{{}^{ABR}_{\ a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{(1-\sigma)s^{\sigma}+\sigma}s^{\sigma}F(s),$

2. $\mathcal{L}\{{}^{ABC}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{(1-\sigma)s^{\sigma}+\sigma}\left(s^{\sigma}F(s) - s^{\sigma-1}f(0)\right),$
3. $\mathcal{L}\{{}^{CFR}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{(1-\sigma)s+\sigma}sF(s),$
4. $\mathcal{L}\{{}^{CFC}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{(1-\sigma)s+\sigma}\left(sF(s) - f(0)\right).$

B. σ −Laplace transform [10]

$$\mathcal{L}_{\sigma}\{f(t)\} = \int_0^{\infty} f(t)e^{-s^{\frac{1}{\sigma}}t}\,dt = F_{\sigma}(s), \qquad s > 0, \sigma \in R_0^+$$

1. $\mathcal{L}_{\sigma}\{{}^{ABR}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{(1-\sigma)s+\sigma}sF_{\sigma}(s),$
2. $\mathcal{L}_{\sigma}\{{}^{ABC}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{(1-\sigma)s+\sigma}\left(sF_{\sigma}(s) - s^{\frac{\sigma-1}{\sigma}}f(0)\right),$
3. $\mathcal{L}_{\sigma}\{{}^{CFR}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{(1-\sigma)s^{1/\sigma}+\sigma}s^{1/\sigma}F(s),$
4. $\mathcal{L}_{\sigma}\{{}^{CFC}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{(1-\sigma)s^{1/\sigma}+\sigma}\left(s^{1/\sigma}F(s) - f(0)\right).$

C. Sumudu transform [11]

$$\mathcal{S}\{f(t)\} = \int_0^{\infty} f(st)e^{-t}\,dt = G(s), \qquad s > 0,$$

1. $\mathcal{S}\{{}^{ABR}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{\sigma}}G(s),$
2. $\mathcal{S}\{{}^{ABC}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{\sigma}}\left(G(s) - f(0)\right),$
3. $\mathcal{S}\{{}^{CFR}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s}G(s),$
4. $\mathcal{S}\{{}^{CFC}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s}\left(G(s) - f(0)\right).$

D. Elzaki transform [12]

$$E\{f(t)\} = s\int_0^{\infty} f(t)e^{-\frac{t}{s}}\,dt = T(s), \qquad s > 0,$$

1. $\mathrm{E}\{{}^{ABR}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{-\sigma}}T(s),$
2. $\mathrm{E}\{{}^{ABC}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{-\sigma}}\left(T(s) - f(0)\right),$
3. $\mathrm{E}\{{}^{CFR}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{-1}}T(s),$
4. $\mathrm{E}\{{}^{CFC}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{-1}}\left(T(s) - f(0)\right).$

E. Natural transform [13]

$$\mathcal{N}\{f(t)\} = \int_0^{\infty} f(ut)e^{-st}\,dt = R(s,u), \qquad s, u > 0,$$

1. $\mathcal{N}\{{}^{ABR}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma\left(\frac{u}{s}\right)^{\sigma}}R(s,u),$
2. $\mathcal{N}\{{}^{ABC}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma\left(\frac{u}{s}\right)^{\sigma}}\left(R(s,u) - \frac{u^2}{s}f(0)\right),$
3. $\mathcal{N}\{{}^{ABR}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma\left(\frac{u}{s}\right)}R(s,u),$
4. $\mathcal{N}\{{}^{ABC}_{a}\mathfrak{D}^{\sigma}_{t}f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma\left(\frac{u}{s}\right)}\left(R(s,u) - \frac{u^2}{s}f(0)\right),$

F. Shehu transform [14]

$$\mathbb{H}\{f(t)\} = \int_0^\infty f(t)e^{-\frac{st}{u}}\,dt = V(s,u), \qquad s,u > 0,$$

1. $\mathbb{H}\{{}^{ABR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma\left(\frac{u}{s}\right)^{\sigma}} V(s,u),$
2. $\mathbb{H}\{{}^{ABC}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma\left(\frac{u}{s}\right)^{\sigma}} \left(V(s,u) - \frac{u}{s} f(0)\right),$
3. $\mathbb{H}\{{}^{ABR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma\left(\frac{u}{s}\right)} V(s,u),$
4. $\mathbb{H}\{{}^{ABC}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma\left(\frac{u}{s}\right)} \left(V(s,u) - \frac{u}{s} f(0)\right),$

G. Mohand transform [15]

$$M\{f(t)\} = s^2 \int_0^\infty f(t)e^{-st}\,dt = \mathcal{M}(s), \qquad s > 0,$$

1. $\mathrm{M}\{{}^{ABR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{-\sigma}} \mathcal{M}(s),$
2. $\mathrm{M}\{{}^{ABC}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{-\sigma}} \left(\mathcal{M}(s) - s f(0)\right),$
3. $M\{{}^{CFR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{-1}} \mathcal{M}(s),$
4. $\mathrm{M}\{{}^{CFC}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{-1}} \left(\mathcal{M}(s) - s f(0)\right).$

H. Jafari transform [16]

$$J\{f(t)\} = p(s) \int_0^\infty f(t)e^{-q(s)t}\,dt = \mathcal{J}(s), \qquad s > 0,$$

1. $\mathrm{J}\{{}^{ABR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma q(s)^{-\sigma}} \mathcal{J}(s),$
2. $\mathrm{J}\{{}^{ABC}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma q(s)^{-\sigma}} \left(\mathcal{J}(s) - \frac{p(s)}{q(s)} f(0)\right),$
3. $\mathrm{J}\{{}^{CFR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma q(s)^{-1}} \mathcal{J}(s),$
4. $\mathrm{J}\{{}^{CFR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma q(s)^{-1}} \left(\mathcal{J}(s) - \frac{p(s)}{q(s)} f(0)\right).$

I. Kamal transform [17]

$$K\{f(t)\} = \int_0^\infty f(t)e^{-\frac{t}{s}}\,dt = \mathcal{K}(s), \qquad s > 0,$$

1. $\mathrm{K}\{{}^{ABR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{\sigma}} \mathcal{K}(s),$
2. $\mathrm{K}\{{}^{ABC}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{\sigma}} \left(\mathcal{K}(s) - s f(0)\right),$
3. $\mathrm{K}\{{}^{CFR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s} \mathcal{K}(s),$
4. $\mathrm{K}\{{}^{CFC}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s} \left(\mathcal{K}(s) - s f(0)\right).$

J. Aboodh transform [18]

$$A\{f(t)\} = s^{-1}\int_0^\infty f(t)e^{-t}\,dt = \mathcal{A}(s), \qquad s > 0,$$

1. $\mathrm{A}\{{}^{ABR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \mathfrak{B}(\sigma)\mathcal{A}(s)$,
2. $\mathrm{A}\{{}^{ABC}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \mathfrak{B}(\sigma)\left(\mathcal{A}(s) - \frac{f(0)}{s}\right)$,
3. $\mathrm{A}\{{}^{CFR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \mathfrak{B}(\sigma)\mathcal{A}(s)$,
4. $\mathrm{A}\{{}^{CFC}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \mathfrak{B}(\sigma)\left(\mathcal{A}(s) - \frac{f(0)}{s}\right)$.

K. Pourreza transform [19]

$$P\{f(t)\} = s\int_0^\infty f(t)e^{-s^2 t}\,dt = \mathcal{P}(s), \qquad s > 0,$$

1. $\mathrm{P}\{{}^{ABR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{-2\sigma}}\mathcal{P}(s)$,
2. $\mathrm{P}\{{}^{ABC}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{-2\sigma}}\left(\mathcal{P}(s) - \frac{f(0)}{s}\right)$,
3. $P\{{}^{CFR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{-2}}\mathcal{P}(s)$,
4. $\mathrm{P}\{{}^{CFC}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{-2}}\left(\mathcal{P}(s) - \frac{f(0)}{s}\right)$.

L. Sawi transform [20]

$$Sa\{f(t)\} = s\int_0^\infty f(t)e^{-s^2 t}\,dt = \mathcal{S}a(s), \qquad s > 0,$$

1. $Sa\{{}^{ABR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{2\sigma}}\mathcal{S}a(s)$,
2. $Sa\{{}^{ABC}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{2\sigma}}\left(\mathcal{S}a(s) - \frac{f(0)}{s}\right)$,
3. $Sa\{{}^{CFR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{2}}\mathcal{S}a(s)$,
4. $Sa\{{}^{CFC}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{2}}\left(\mathcal{S}a(s) - \frac{f(0)}{s}\right)$.

M. G-transform [21]

$$\mathcal{G}\{f(t)\} = s\int_0^\infty f(t)e^{-s^2 t}\,dt = \mathcal{G}(s), \qquad s > 0, \sigma \in R_0^+$$

1. $\mathcal{G}\{{}^{ABR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{\sigma}}\mathcal{G}(s)$,
2. $\mathcal{G}\{{}^{ABC}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s^{\sigma}}\left(\mathcal{G}(s) - s^{\sigma-1}f(0)\right)$,
3. $\mathcal{G}\{{}^{CFR}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s}\mathcal{G}(s)$,
4. $\mathcal{G}\{{}^{CFC}_{a}\mathfrak{D}^{\sigma}_{t} f(t)\} = \frac{\mathfrak{B}(\sigma)}{1-\sigma+\sigma s}\left(\mathcal{G}(s) - s^{\sigma-1}f(0)\right)$.

Conclusions

Fractional calculus is one of the most important topics that occupy researchers and scientists in various mathematical, engineering, physical, and other disciplines. It is known that integral transformations are capable of solving differential equations, so the presentation of this paper was very necessary in order for researchers to be able to obtain the integral transformations of fractional calculus in one paper, and for that this manuscript was presented.

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