Quadrature And Cubatura Formulas for Calculation of Exact Integrals

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ABSTRACT

The article presents the basic quadrature and cubature formulas used to calculate definite integrals. Quadratic formulas are relatively simple, therefore, theoretical information about them is given. Since the cubature formulas are a little more complicated than the quadrature formulas, the calculation of definite integrals using the same formulas is shown and their degree of accuracy is compared. Simpson's method has been shown to give relatively accurate results.

Keywords: cubic formulas, quadrature formulas, Simpson method, central rectangle method, error, relative error.

Introduction:

Theoretical research in the field of computational mathematics is mainly grouped around numerical methods for solving typical mathematical problems. One of the classic problems of this field is to construct the approximate calculation formulas of these integrals.

The numerical values of one-time integrals are called short squares in geometric terms.

Great scientists have often dealt with such issues. For example: Gauss, Chebishev, Euler, Newton and others.

Main Part:

By quadratic formula we mean the following approximate equation:

$$\int_a^b f(x)dx \approx \sum_{i=1}^{n} C_i f(x_i)$$

where is the coefficient of the C_i - quadratic formula, x_i- the nodes of the quadratic formula, the number of N-node points.

In the approximate calculation of integrals, if the function under the integral is a single variable, it is called a quadratic formula, Sard and Nikolsky were the first to deal with such problems. If the function under the integral has two or more variables, it is called a cubic formula, and Sobolev was the first to deal with such problems. That is, cubic formulas are used to calculate multiple integrals.

This requires the calculation of the following integral:

$$\iint_\sigma f(x,y)dxdy \qquad (1)$$

Here the function f(x,y) is defined in the field σ and is continuous. Then we can define a set of nodes that belong to this field.

This integral is calculated by the following approximate formula:

$$\iint_\sigma f(x,y)dxdy \approx \sum_{i=1}^{n} A_i f(x_i, y_i) \qquad (1`)$$

where A_i is the coefficient of the cubicle formula, for all polynomials defined in formula (1`) we require the following cubicle formula:

$P_n(x,y) = \sum_{k+l}^{n} c_{kl} x^k y^l$ (2)

in which case the polynomial does not exceed a given number n.

In order for the above conditions to be met, the formula (1`) must be sufficient and accurate to calculate the $x^k y^l$

Assuming that f(x,y)= $x^k y^l$ in the formula (1`), we have the following formula:

$I_{kl} = \iint_\sigma x^k y^l dxdy = \sum_{i=1}^{n} A_i x_i^k y_i^l \quad (k,l = 0,1,\dots.n,\ k+1 < n)$ (3)

thus the coefficients A_i in formula (1) can be determined from the system of linear equations (3) in general. For the system (3) to be in a definite state, the number of unknowns must be equal to the number of equations n. Here we determine the grid values of the indicators:

$$\mathrm{N} = (\mathrm{n}+1) + \mathrm{n} + \dots + 1 = \frac{(n+1)(n+2)}{2}$$

This formula can be further clarified as follows:

$$\iint_\sigma f(x,y)dxdy \approx \sum_{i=1}^{n} A_i B_{ij} f(x_i, y_j)$$

Here AA_i and B_{ij} are real numbers and are the coefficients of the cubic formula.

Other types of method formulas can be derived on the basis of the same formulas.

Let's use the center square method for multiple integrals. Let us be given the integral in (1) (a≤ x ≤b, $c \le y \le d$). If we find the center points for this integral and change the formula for these points, we get the following formula:

$\iint_G f(x,y)dxdy \approx f(\frac{a+b}{2}, \frac{c+d}{2})(b-a)(d-c)$ (4)

To increase the accuracy of the formula, the area G can be divided into cells, and if we use the formula (4) for each cell, that is, if it has a given rectangular shape, then the problem can be discretized as follows:

$$\Delta x_i = x_i - x_{i-1},\ \ \Delta y = y_i - y_{i-1}$$

$$\iint_{G_{ij}} f(x,y)dxdy \approx f(\bar{x}_i, \bar{y}_j)\Delta x_i \Delta y_i$$

$$\bar{x}_i = \frac{x_{i-1} + x_i}{2}\ ,\quad \bar{y} = \frac{y_{i-1} + y_i}{2}$$

assemble all surfaces:

$$\iint_G f(x,y)dxdy = \sum_{i=1}^{M} \sum_{j=1}^{N} f(\bar{x}_i, \bar{y}_j)\, \Delta x_i \Delta y_i$$

we create. Here M is the number of horizontal sections and N is the number of vertical sections (cells).

If we want to increase the accuracy of the results obtained by the method, we need to increase the number of N and M segments. But we have to make sure that the value of the M/N ratio is constant.

Consider a similar Simpson formula. Let the area to be integrated be a rectangle (G:a≤ x ≤A, $b \le y \le B$). Let the sides of the sphere be parallel to the coordinate axes. We're going to divide this area into two and

$h = \frac{A-a}{2},\quad k = \frac{B-b}{2}$

we find that We get 9 points in total. We use the following formula to calculate multiple integrals:

$$\iint_\sigma f(x,y)dxdy \approx \int_a^A dx \int_b^B f(x,y)dy$$

Calculating the internal integral using the Simpson formula, we obtain:

$$\int_\sigma \int f(x,y)dxdy \approx k/3\left[\int_a^A f(x,y_0)dx + 4\int_a^A f(x,y_1)dx + \int_a^A f(x,y_2)dx\right]$$

Now we re-apply the Simpson formula to each integral and find:

$\iint_\sigma f(x,y)dxdy \approx kh/9\big[[f(x_0,y_0) + f(x_2,y_0) + f(x_0,y_2) + f(x_2,y_2)\,] + 4[f(x_1,y_0) + f(x_0,y_1) + f(x_2,y_1) + f(x_1,y_2)] + 16f(x_1,y_1)\big]$ (5)

this formula is called Simpson's cubicle formula.

For convenience, if we enter the notation $f(x_i,y_j) = f_{ij}$ we get the following final formula:

$$\iint_\sigma f(x,y)dxdy = \frac{hk}{9}\sum_{i=0}^{2n}\sum_{j=0}^{2m}\lambda_{ij}f_{ij} \quad (6)$$

Here λ_{ij} corresponds to the following matrix elements, respectively:

$$\lambda_{ij} = \begin{pmatrix} 1 & 4 & 2 & 4 & & 2 & 4 & 1 \\ 4 & 16 & 8 & 16 & \cdots & 8 & 16 & 4 \\ 2 & 8 & 4 & 8 & & 4 & 8 & 2 \\ 4 & 16 & 8 & 16 & & 8 & 16 & 4 \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots \\ 4 & 16 & 8 & 16 & & 8 & 16 & 4 \\ 2 & 8 & 4 & 8 & & 4 & 8 & 2 \\ 4 & 16 & 8 & 16 & \cdots & 8 & 16 & 4 \\ 1 & 4 & 2 & 4 & & 2 & 4 & 1 \end{pmatrix}$$

If the integration area is complex, the following auxiliary function can be used:

$$f\,(x,y) = \begin{cases} f(x,y,)\,, & (x,y) \in \sigma; \\ 0, & (x,y) \in \sigma; \end{cases}$$

Using this formula, formula (6) can be written as:

$$\iint_\sigma f(x,y)dxdy = \iint_\sigma f^*(x,y)\text{dxdy}$$

Let's look at an example of a multiple integral using the above formulas and analyze which method is better.

Results And Discussion:

To do this, consider the following integral. Since the aim of our work is to determine which method has the highest degree of accuracy, we choose the integral that can be obtained analytically:

$$\text{I} = \iint_D xydxdy \qquad \text{D}:\{x = 3,\ x = 5,\ 3x - 2y + 4 = 0,\ 3x - 2y + 1 = 0\}$$

First of all, we can calculate this integral analytically and determine the result. (I = 99.5) To calculate this integral using numerical methods, we use the Simpson and Central Rectangle methods. First of all, let's get the result using the method of central rectangles.

First of all, let's find out in which field the integral lies (Figure 1): the barred areas are the areas in which we need to integrate. But using the auxiliary function we get only the corresponding values.

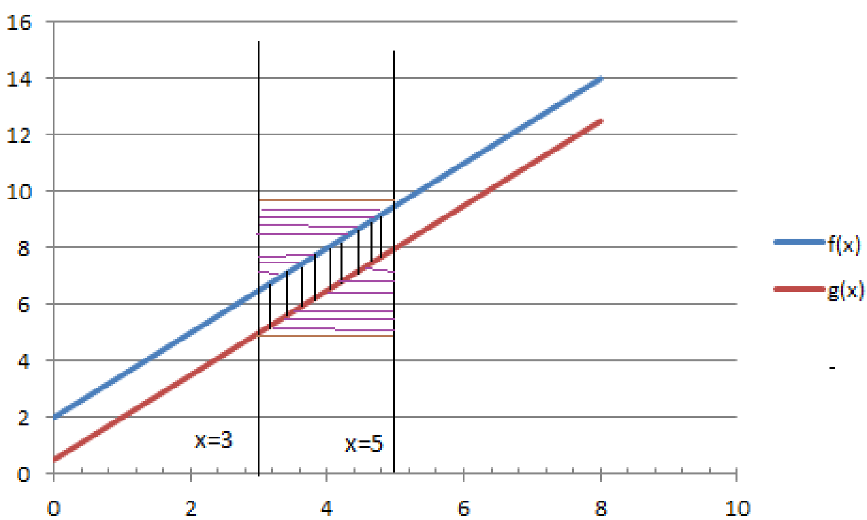

Figure 1. Given D area

As a result, for the method of central rectangles, we obtain the following result, assuming that M=N=4, h=0.5, k=1.125 (of course, further magnification of the values of M and N to make the result more accurate possible):

$$I = \iint_D xydxdy = \sum_{i=1}^{M}\sum_{j=1}^{N} f(\overline{x_i}, \overline{y_j})\Delta x_i \Delta y_j = h \cdot k \begin{bmatrix} f(x_1, y_1) + f(x_1, y_2) + \\ + f(x_2, y_2) + \\ + f(x_3, y_3) + f(x_4, y_3) \end{bmatrix} = 99{,}93164$$

The calculated averages and their function values are given in the following table:

Table 1

Values that the function accepts for intermediate values

	3.25	3.75	4.25	4.75
5.5625	18.07813	20.85938	23.64063	26.42188
6.6875	21.73438	25.07813	28.42188	31.76563
7.8125	25.39063	29.29688	33.20313	37.10938
8.9375	29.04688	33.51563	37.98438	42.45313

Now, from the second method we want to calculate the integral, the Simpson method, we calculate the same integral. Here, too, all of the above initial values are valid. Just keep in mind that the number of sections must be doubled when dividing the intervals into parts:

$$\lambda_{ij} = \begin{pmatrix} 1 & 4 & 2 & 4 & 1 \\ 4 & 16 & 8 & 16 & 4 \\ 2 & 8 & 4 & 8 & 2 \\ 4 & 16 & 8 & 16 & 4 \\ 1 & 4 & 2 & 4 & 1 \end{pmatrix}$$

Knowing that the elements of the λ_{ij} matrix given above are the coefficients in the Simpson formula, we can perform the following calculations:

$$I = \frac{0.5*1.125}{9}\begin{bmatrix} 15+16*21.4375+4*18.375+8*25.375+4*29+ \\ +16*37.6875+4*41.875+1*47.5 \end{bmatrix} = 99{,}5625$$

Now let's compare the results by hand. Notation: I - is the result of the analytical method, I_s is the result of the exact integral using the Simpson method, I_t is the result of the central rectangle. Comparing all the results, we got the following results:

$$|I - I_s| = 6.25\%$$

$$|I - I_t| = 43.164\%$$

In conclusion, we can say that in both methods, the result is almost the same. Both methods require an increase in the number of sections to increase accuracy. However, with relatively similar fragments, the Simpson method is much more accurate and reliable than the "rectangular method for center points."

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