



# Synthesis of a Robust Control System with A Reference Model of a Nonlinear Dynamic Object with State Delay

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## ABSTRACT

In this paper, control problem with a reference model for a nonlinear dynamic object with a delay in state is analyzed and solved. The suggested control algorithm is a special form that links in the opposite direction, where an auxiliary contour and observers are used which is considering all information about the perturbations and nonlinearities of the object. Differently from all known approaches, there were used observers, which allow compensate observed errors from the first filter.

## Keywords:

External perturbation, deviation of the argument, object lag, dynamic processes, transient processes, closed loop, reference model, error of tracking

## Introduction

The modern period of development of control theory is characterized by the formulation and solution of problems that take into account the inaccuracy of our knowledge about control objects and external disturbances acting on them [1]. When creating control systems for dynamic objects, additional difficulties often arise associated with the presence of delay and nonlinearity of the object [2]. In this case, the central problem is the construction of a robust controller that ensures the achievability of the control goal for objects

described as nonlinear with argument deviations.

One of the approaches to solving the problem is to evaluate disturbances added to the delay and nonlinearity using indirect measurements in order to compensate for their influence on the controlled variable.

## Formulation of the problem

Let us consider a control object, the dynamic processes in which are described by the equation

$$Q(p)y(t) = kR(p)u(t) + N(p)y(t - h(t)) + \sum_{i=1}^n \sum_{j=1}^q p^{n-i} \phi_{ij}(y(t))\tau_j + f(t) \quad (1)$$

where  $y(t)$ ,  $u(t)$  - scalar controlled variable and control;  $p = d/dt$  - differentiation operator;  $Q(p)$ ,  $R(p)$  - normalized differential operators;  $\deg Q(p) = n$ ;  $\deg R(p) = m$ ;  $\deg N(p) \leq n - 1$ ; lag time  $h(t)$  - limited function;  $k > 0$ ,  $\phi_{ij}$  ( $1 \leq i \leq n, 1 \leq j \leq q$ ) - smooth functions satisfying Lipschitz;  $\tau_j$  - unknown constants;  $f(t)$  - external disturbance.

The required quality of transient processes in an object is specified by the equation of the reference model

$$Q_m(p)y_m(t) = k_m g(t) \quad (2)$$

where  $g(t)$  - scalar limited reference;  $k_m > 0$ ;  $y_m(t)$  - limited scalar output,  $\deg Q_m(p) = n - m$ .

The designed control system must ensure the fulfillment of the target condition

$$|y(t) - y_m(t)| < \delta \quad \text{при } t > T \quad (3)$$

where  $\delta$  - some fairly small number,  $T > 0$ .

Let us consider a control object, the dynamic processes in which are described by the equation

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Here  $y_i(s)$  - continuous, bounded initial functions;  $\phi_{ij}(y(t)) (1 \leq i \leq n, 1 \leq j \leq q)$  - smooth functions satisfying the Lipschitz condition;  $\tau_j$  - unknown constants;  $Q(p), R(p), N(p), M(p)$  - operators;  $\xi \in E$  - unknown parameters;  $E$  - known set of possible values  $\xi$ ;  $k = f(\sigma)$ ;  $h(t)$  - lag time;

$\frac{dh(t)}{dt} < 1$ ;  $h(t) > 0$ ;  $g(t)$  and  $f(t)$  setting and disturbing influences, respectively;

$\deg Q(p) = n$ ;  $\deg K(p) = m$ ;  $\deg N(p) = n - 1$  operator orders  $Q(p)$  and  $R(p)$ ;  $\deg Q_n(p) = n - m$  operator orders;

Let's imagine the operators  $Q(p)$  and  $R(p)$  as  $Q(p) = Q_m(p) + \Delta Q(p)$ ,  $R(p) = R_m(p) + \Delta R(p)$  where  $Q_m(p), R_m(p)$  - operators with known coefficients such that the polynomials  $Q_m(p), R_m(p)$  are Hurwitz and have orders  $n$ , respectively. Then we transform (4) into an equivalent equation for the output  $y(t)$ :

$$Q_m(p)y(t) = kR_m(p)\left(u(t) + \frac{\Delta R(p)}{R_m(p)}u(t) + \frac{\Delta Q(p)}{kR_m(p)}y(t) + \frac{N(p)}{kR_m(p)}y(t - h(t)) + \frac{1}{kR_m(p)}\sum_{i=1}^n \sum_{j=1}^q p^{n-i} \phi_{ij}(y(t))\tau_j + \frac{1}{kR_m(p)}f(t)\right) \quad (5)$$

Let's create an equation for the error  $e(t) = y(t) - y_m(t)$ , subtracting (2) from (5):

$$Q_m(p)e(t) = kR_m(p)\left(u(t) + \frac{\Delta R(p)}{R_m(p)}u(t) - \frac{\Delta Q(p)}{kR_m(p)}y(t) + \frac{N(p)}{kR_m(p)}y(t - h(t)) + \frac{1}{kR_m(p)}\sum_{i=1}^n \sum_{j=1}^q p^{n-i} \phi_{ij}(y(t))\tau_j - \frac{1}{kR_m(p)}f(t) - \frac{k_m g(t)}{kR_m(p)}\right) \quad (6)$$

If it is possible to measure the derivatives of the control action, we will set the control law in the form

$$u(t) = T(p)v(t) \quad (7)$$

Then control (6) will take the following form:

$$Q_m(p)e(t) = kT(p)(v(t) + \frac{\Delta R(p)}{R_m(t)}v(t) - \frac{\Delta Q(p)}{kR_m(p)T}y(t) + \frac{N(p)}{kR_m(p)T}y(t-h(t)) + \frac{1}{kR_m(p)T} \sum_{i=1}^n \sum_{j=1}^q p^{n-i} \varphi_{ij}(y(t))\tau_j + \frac{1}{kR_m(p)T}f(t) - \frac{k_m g(t)}{kR_m(p)} \tag{8}$$

If it is impossible to measure the derivatives of the control action  $v(t)$ , we will define the control law in the form

$$u(t) = T(p)\bar{v}(t) \tag{9}$$

where  $\bar{v}(t)$  - signal assessment obtained from the observer [3].

$$\xi = F_0\xi(t) + B_0(v(t) - \bar{v}(t)), \bar{v}(t) = L\xi(t) \tag{10}$$

Substituting (7) and (6), we obtain the equation

$$Q_m(p)e(t) = \beta T(p)v(t) + \bar{\varphi}(t) + \beta T(p)(\bar{v}(t) - v(t)) \tag{11}$$

where

$$\begin{aligned} \bar{\varphi}(t) = & (k - \beta)v(t) + k \frac{\Delta R(p)}{R_m(p)}v(t) - \frac{\Delta Q(p)}{R_m(p)T}y(t-h(t)) + \frac{1}{R_m(p)T} \sum_{i=1}^n \sum_{j=1}^q p^{n-i} \varphi_{ij}(y(t))\tau_j + \\ & + \frac{M(p)}{R_m(p)T}f(t) - \frac{k_m}{R_m(p)T}g(t) \end{aligned}$$

Let us choose the polynomial  $T(\lambda)$  so that the transfer function

$$\frac{T(\lambda)R_m(\lambda)}{Q_m(\lambda)} = \frac{1}{\lambda + a_m}$$

Then equation (11) is transformed to the form

$$(p + a_m)y(t) = \beta v(t) + \varphi(t) \tag{12}$$

where

$$\varphi(t) = \frac{1}{T(p)R_m(p)}\bar{\varphi}(t) + \beta(\bar{v}(t) - v(t))$$

The signal  $\varphi(\lambda)$  concentrated all the uncertainty of the parameters of the control object and external disturbances.

Let's introduce an auxiliary contour:

$$(p + a_m)y(t) = \beta v(t) \tag{13}$$

and, taking into account (12), (13), we create an equation for the mismatch  $\xi(t) = y(t) - \bar{y}(t)$ :

$$(p + a_m)y(t) = \varphi(t)$$

Thus, if  $n - m - 1$  derivatives of the signal  $v(t)$  and the first derivative of the controlled variable  $e(t)$  are available for measurement, then, having formed  $v(t)$  in the form

$$v(t) = -\frac{1}{\beta}(p + a_m)\xi(t) \tag{14}$$

We obtain that the control law (7), (14) ensures the asymptotic stability of the system (4), (7), (14) with respect to the variable  $e(t)$ , and the equation of the closed-loop system will have the form

$$(p + a_m)y(t) = 0$$

If it is impossible to measure the necessary signals, instead of (14), the signal  $v(t)$  is formed in the form

$$v(t) = -\frac{1}{\beta}(p + a_m)\bar{\xi}(t)$$

where  $\bar{\xi}$  - assessment obtained from the observer

$$\begin{aligned} \dot{z} &= \bar{F}_0 z(t) + B_0(\xi(t) - \bar{\xi}(t)) \\ \bar{\xi}(t) &= L_2 z(t) \end{aligned} \tag{15}$$

Thus, the equations of the closed system have the following form (16).

Control object:

$$Q(p)y(t) = kR(p)u(t) + N(p)y(t - h(t)) \frac{1}{kR_m(p)T} \sum_{i=1}^n \sum_{j=1}^q p^{n-i} \varphi_{ij}(y(t))\tau_j + f(t)$$

Control Law:  $u(t) = T(p)v(t), u(t) = T(p)\bar{v}(t)$

Observer 1:

$$\begin{aligned} \xi &= \bar{F}\xi(t) + \bar{B}_0(v(t) - \bar{v}(t)), \\ \bar{v}(t) &= L\xi(t) \end{aligned} \tag{16}$$

Auxiliary circuit:  $(p + a_m)\bar{y}(t) = \beta v(t),$

$$v(t) = -\frac{1}{\beta}(p + a_m)\xi(t), v(t) = -\frac{1}{\beta}(a_m + p)\bar{\xi}(t) \tag{17}$$

Observer 2:

$$\begin{aligned} \dot{z} &= \bar{F}_0 z(t) + B_0(\xi(t) - \bar{\xi}(t)) \\ \xi(t) &= L_2 z(t). \end{aligned} \tag{18}$$

**Example.** Let the control object be described by a nonlinear equation with a deviating argument

$p^4 + a_1 p^3 + a_2 p^2 + a_3 p + a_4)y(t) = (b_0 p + b_1)u + (c_1 p^3 + c_2 p^2 + c_3)f(t) + (d_1 p^3 + d_2 p^2 + d_3 p + d_4)\varphi_1(y(t)) + (n_1 p^3 + n_2 p^2 + n_3 p + n_4)y(t - h) + (m_1 p^3 + m_2 p^2 + m_3 p + m_4)\varphi_2(y(t))$  The uncertainty class is given by the inequalities  $-7 \leq a_i \leq 7, 20 \leq b_i \leq 50; -2 \leq c_k \leq 8; |f(t)| < 1.$

The reference model equation has the form  $(p + 3)^3 y_m(t) = 81r(t)$ . Let's choose a polynomial  $T(\lambda) = \lambda^2 + 16\lambda + 64, \beta = 20$ . Control influences are formed in the form:

$$U(t) = 9\xi_1(t) + 6\xi_2(t) + \xi_2(t).$$

$$v(t) = \frac{1}{20}(8\xi(t) + \dot{z}(t))$$

After conducting a simulation experiment in the MATLAB environment, analysis of the simulation results of this control object allows us to conclude that the worst accuracy  $\delta = 0,015$  in terms of tracking error  $c(t)$  is achieved at  $a_j = \inf a_i(\xi) = -7, b_0 = \inf k(\xi) = 20$ .

Thus, for the object considered in this example, the proposed scheme for generating a control action ensures the fulfillment of control goal (3) with a tracking error  $e(t) = y(t) - y_m(t) = 0.002$ .

## Conclusion

The problem of nonlinearity of the control object in the presence of delay arises with a more detailed study of the nature of the controlled dynamic processes. Therefore, it is important that the given scheme for generating a control action allows not only to track a given reference signal with a given accuracy, but also to take into account the delay effect, the presence of nonlinearity under conditions of a priori uncertainty and the action of external disturbances.

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