A review of the evolution of the stress equations arising in the brackets of ordinary concrete

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ABSTRACT

The text discusses the introduction of corbels or brackets, which are structural features that extend from a wall or column to support a weight. Corbels are commonly used in precast buildings to support prefabricated beams on columns. They are different from deep beams and are considered as mechanisms of shear transfer. Traditional design approaches involve using horizontal stirrups to enhance the shear capacity of corbels and prevent failure. The text mentions various researchers who have studied the behavior and strength of reinforced concrete corbels. Franz and Niedenhoff presented a truss analogy for corbel construction. Kriz and Raths conducted tests on corbels with different variables such as breadth, effective depth, reinforcing ratio, and concrete strength. They found that the strength of corbels is directly related to the strength of the concrete and the primary reinforcement ratio. The text also discusses the Mast shear-friction theory, which is based on the concept of frictional resistance along cracks in concrete. The theory considers the reinforcement that spans a crack to provide frictional resistance and undergo yielding due to strain. Hermansen and Cowan proposed a modified shear-friction theory that includes the coefficient of friction and apparent cohesive stresses. Overall, the text provides an overview of corbels, their behavior, and different theories and approaches related to their design and strength.

Keywords: Corbels or brackets

Introduction:

The structural features known as corbels or brackets extend from a wall or column to support a weight. The name "corbel" refers to cantilevers have a shear span-to-depth ratio (av/d) of 1 or less [1]. They are often built alongside a column or wall. In precast buildings, corbels are widely employed to support prefabricated beams on columns. Although "corbel" and "bracket" are frequently used interchangeably [2], corbels are still referred to as such when they protrude from walls rather than columns. Corbels are not flexural members intended for shear in accordance with ACI-318M but rather simple trusses or deep beams [3]. The typical reinforcement concrete corbel is shown in Figure 1.

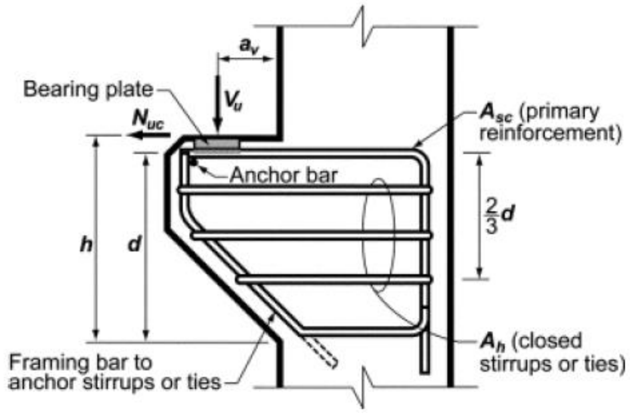

Fig. (1) – The present study concerns the loads and reinforcement of a typical reinforced concrete corbel.

The corbel's stress state becomes two-dimensional due to the tiny (av/d) ratio, and shear deformations can affect its nonlinear stress behavior even when the corbel is in an elastic condition. Shear strength thus assumes importance. Because they transfer potentially considerable horizontal stresses from the supporting beam to the corbel, corbels differ from deep beams [4]. As a result, reinforced concrete corbels are frequently thought of as Mechanisms of shear transfer. Traditional design approaches necessitate the incorporation of horizontal stirrups across the entire depth of corbels to enhance their shear capacity and avert potential catastrophic failure. Examples of these failure mechanisms include diagonal splitting failure mechanisms. The use of just horizontal stirrups is inappropriate because diagonal tension fractures are less steeply inclined when (av/d) is greater than unity. If (av/d) is less than 2, ACI-318 advises constructing the corbel using the strut-and-tie models in Chapter 23 [3]. On the top surface of corbels, steel bearing plates or angles are frequently used to provide a consistent contact surface and disperse the response.

Under monotonic loading, normal-weight reinforced concrete corbels:

Numerous researchers have conducted in-depth studies on the behavior and strength of reinforced concrete corbels with primary and secondary reinforcement. Investigations have been conducted into a number of factors influencing corbel behavior. Franz and Niedenhoff carried out some of the early research in this area in 1963 [8], presenting the most basic truss analogy for corbel construction. As shown in Fig. (2), there are consider the corbel a straightforward strut-and-tie system that was subjected to an external force V. The slightly sloped tensile force, Ft, was believed to be horizontal for design purposes. Equation 1 was used to determine this force.

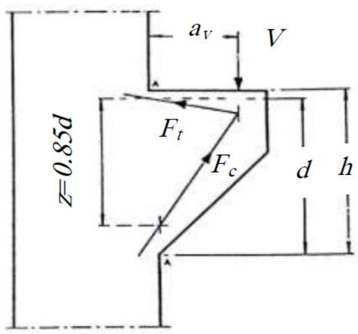

Fig. (2) - Franz and Niedenhoff's straightforward truss analogy for designing concrete corbels

$$Ft = \left(\frac{v \cdot a_U}{z}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots \mathbf{1}$$

In equation (2), the size, in mm2, of the major steel area, required to resist the force (V) in kN,

$$A = \frac{\mathrm{Ft}}{\mathrm{fs(all)}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \mathbf{2}$$

is computed in relation to the MPa allowed tensile reinforcement stress (fs all). The tensile force, Ft, was calculated as 0.85 d, where (d) is efficient depth of the corbel in millimeters. According to Franz and Niedenhoff's [8] suggestion, the major tension reinforcement might be fastened to the corbel's exterior face using horizontal loops. Aside from being uncommon cases when the loads are applied to the bottom of the corbels, Using heavy bars that are inclined to a longitudinal axis's of the support an element is frequently improper and uneconomical, they further suggested. Despite the fact that several of their suggestions involved diagonal links, they represented a major advancement over earlier conventional approaches that depended in inclined reinforcement.

Kriz and Raths [6] tested reinforcement concrete corbels in three sets. The initial groups of testing. was exploratory, the second set solely featured vertical loads for corbels; and the third set combined vertical and horizontal load for corbel combined vertical and horizontal loads for corbels. To create testing protocols and reinforce details, exploratory tests were carried out.

Corbels' strength and behavior vary depending on a number of variables. A corbel's breadth (b), depth's effective (d), reinforcing ratio (ρ), concrete strengthen (fc') and the (av/d) ratios all contribute to its ultimate strength (Vu). According to the exploratory tests, the strength of the corbel is directly related to the strength of the concrete ('cf), and it is unaffected by the loads carry by the column or the placement and quantity of reinforcement on the columns. Strength increase with an increases in the primary reinforcement ratio and decrease with an increase in the (av/d) ratio. Additionally, the study demonstrated that stirrups, which are horizontal reinforcement, are just as efficient as main tension reinforcement in resisting vertical stresses.

Kriz and Raths conducted further testing to look at how corbels might be affected by combining vertical and horizontal loading. According to the findings, stirrups do not make corbels more resistant to combined loading than corbels that are simply subjected to vertical stresses. Therefore, any strength provided by stirrups should be considered a reserve. As a result, it was suggested that you always give a minimum number of stirrups. Based on their comprehensive test data fitting curves, Kriz and Raths provided an equation for determining the maximum strength achievable of corbels exposed to vertical or mixed loads.

$$V_u = \phi \cdot \boldsymbol{b} \cdot \boldsymbol{d} \cdot \sqrt{f_c'} \cdot F_1 \cdot F_3 \qquad \ldots\ldots\ldots \mathbf{3}$$

$$F_1 = 6.5\left(1 - 0.5^{\frac{d}{a}}\right) \qquad \ldots\ldots\ldots\ldots \mathbf{3a}$$

$$F_3 = \frac{(1000 \cdot \rho)^{\left(\frac{1}{3} + \frac{0.4H}{V}\right)}}{10} \qquad \ldots\ldots\ldots\ldots. \mathbf{3b}$$

$\rho = \frac{A_s + A_h}{b \cdot d} \leq 0.]02$, for corbel subject to vertical load only. …… 3c

$\rho = \frac{As}{b \cdot d} < 0.013$, for corbel subject to combined load. ……… 3d

? The ratio of reinforcement in the column face.

$\boldsymbol{\phi}$ Reduced capacity is a factor; as shear primarily governs corbel and bracket behavior, the single values of (ϕ = 0.85) are necessary to every design circumstances.

$\boldsymbol{H/V}$ Load ratio from horizontal to vertical direction.

$\boldsymbol{A_s}$ The area of the tension reinforcement, measured in square millimeters (mm^2).

$\boldsymbol{A_h}$ the area of close stirrup, mm^2, $\boldsymbol{A_h}$ cannot be lower than $\boldsymbol{A_s / 2}$.

A design formula developed by Kriz and Raths had been included in the ACI 318-1971 [10] and "PCI Design Handbook"-1972 [11] for corbels

(Mast 1968) The technique has been introduced for the design of concrete connections utilizing physical models that are founded on the shear-friction hypotheses. The present methodology was originally employed for the purpose of devising interface connections in composite beams. Subsequently, it was expanded to encompass concrete corbels, drawing upon empirical data obtained from experiments conducted by **Kriz and Raths** in 1965. The theory of shear friction is a straightforward concept that can be readily conceptualized, as illustrated in Figure 2. Mast's postulations pertained to a concrete specimen that has undergone a state of fissuring and was subjected to a compressive force perpendicular to the crack as well as the shear force parallel to the cracks. The shearing forces can be impeded by the friction occurring along the cracks. In the event that reinforcement is administered at a perpendicular angle to the crack, the concrete's slippage and detachment will exert pressure on the steel tension. In this scenario, the reinforcement will function as a tension element in lieu of a dowel.

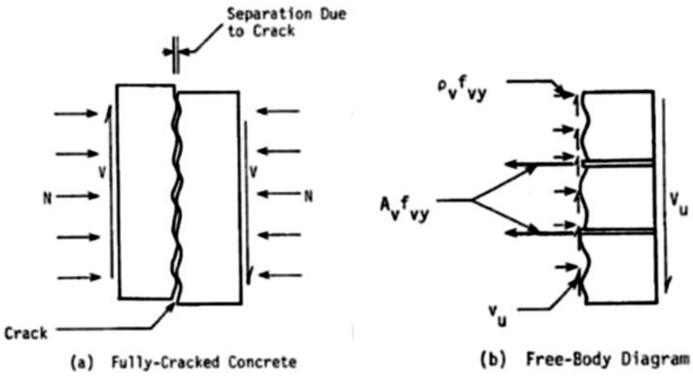

Fig. (2) - **Base for Mast Shear-Friction Theory**

The generation of a tensile force results in the development of a counteracting compressive stress between the concrete surfaces situated on either side of the fracture. As depicted in Figure (2.b), the interface pressure attains its maximum value at (Avf fy), By applying the principle of equilibrium, where the sum of horizontal forces is set to zero, the coefficient of friction (μ) multiplied by the normal force can be used to quantify The ability of concrete to resist sliding forces. In this context, (Avf) represents the aggregate steel area that traverses a cracks, and (fy) indicates the yield strengthing.

$$V_n = A_{vf} \tan(\alpha) f_y \quad \text{...... 4}$$

Where (μ) is equal to tan (α), with α representing an angle of internal frictions. A reinforcement ratio (ρ) is defined as (Avf / Ac) where (Ac) is the area of a cracking surfaces. Equation (4) can be write as follows:

$$v_n = \rho f_y \tan(\alpha) \quad \text{... 5}$$

Where,

$$v_n = \frac{V_n}{A_C} \quad \text{......5 a}$$

Vn represents the shear forces at ultimate loads, measured in kilo newton (kN).

vn represents the shear stress at ultimate loads, measured in (MPa).
Avf represents a total steel area cross the cracks, which has a yield strengths of (fy). The values of (μ = tanα) were determined from tests.
Table) 1), as provided by Mast, was utilized for design purposes and applied to the tests data gathered by **Kriz and Raths**. Mast specifically examined specimens with (av/d) ratios that were less than or equal to 0.7 and were subjected to steel yielding. This was based in the assumption that to av/d ratios exceeding this threshold, a required amount of steel would be governed by flexure rather than shear. In cases of combine load the nominal shear stress was computed in the following manner:

$$V_n = \left(\infty \cdot fy - \frac{H}{bn}\right)\tan\alpha \qquad \ldots\ldots 6$$

Where H represents the external horizontal forces at ultimate loads.

Table (1) - Value of as given by Mast [11]

Types of Surfaces	*tanα*
Rough surfaces, fracture in monolithic concrete, concrete to concrete.	1.4
Concrete to concrete, a smooth finish	0.7
steel to Concrete combined beam	1

According to the modified shear-friction theory proposed by Hermansen and Cowan in 1974, when a fracture occurs in the shear plane, the reinforcement that spans a crack not only provides frictional resistance to prevent movement of concrete along the crack but also undergoes yielding due to strain.

$$v \; c \; f_y \; tan \qquad \ldots\ldots 8$$

In the modified shear-friction theory proposed by Hermansen and Cowan, the presence of the coefficient of friction (τανα) and apparent cohesive stresses (c) are considered. They suggest using a value of (c) as 4 MPa and (τανα) as 0.8 for achieving a safe design.
Hermansen and Cowan's main finding is that there is no significant difference in behavior between specimens with a single corbel (exterior corbel) and those with a double corbel (interior corbel).
Mast (1968) and Hermansen and Cowan (1974) have suggested modified shear-friction theories. The values of the tangent of the angle of internal friction (tanα) can vary between these theories. When the steel crossing the direct shear plane is minimal and the product of the reinforcement ratio is high, the strength reduction factor and the confinement factor (ρφψ) are low, and assuming that the cohesion (c) is zero, Mast's proposal shall be more cautious compared to the equation (8) propose by Hermansen and Cowan (1974). So for higher magnitudes of the product of density and porosity, this pattern will be inverted.
The ACI-318 building code has adopted a design procedure for corbels proposed by Mattock in 1976. The procedure is applicable to corbel with a (av/d) ratio of unity or less under vertical and horizontal stresses. Mattock designs using a flexural model. Which is a straightforward mechanical model. **(Mattock, 1976)** [11]. the primary methodology for designing involves:

1) Ultimate shears stresses (vu) should not exceed a value indicated in the equation **Eq 9**

$$v_u \le \begin{cases} 0.2 \cdot f_c' \ MPa \\ 5.5 \ MPa \end{cases} \qquad \ldots\ldots\ldots\ldots 9$$

2) The reinforced area, Avf, cross the shears plane needs to be calculate accord to the equation. **Eq 10** Where:

$$A_{vf} = V_u / (\phi \cdot f_y \cdot \mu) \quad \text{.................................} \mathbf{10}$$

ϕ = **0.85.** μ = **1.4, for** corbel cast monolithically with the columns.

3) Calculate the ultimate moments and the corbel-column interfaces must resist equation (11):

$$M_u = V_u \cdot a_v + N_u (h - d) \quad \text{......} \mathbf{11}$$

4) calculate the reinforcement area, Af, needed to resist the ultimate moments calculate in the previous items.

$$A_f = M_u / (\phi f_y (d - a/2)) \quad \text{..........................} \mathbf{12}$$

5) Calculate At, the reinforcement need to resist Nu, the horizontal forces.

$$A_t = N_u / (\phi f_y) \quad \text{................} \mathbf{13}$$

6) The area of primary tension reinforcement, As, will be a bigger value of any of a following:

$$A_s = \begin{cases} Af + At \\ (2/3Avf) + At \end{cases}$$

7) Closed stirrup with area, Ah, parallel to the primary tension reinforcement, As, must be equally distributed within two-thirds of the effective depth, as shown in Eq. (15):

$$A_h = 0.5 (A_s - A_t)$$

8) The steel ratios $(\rho = A/bd)$ must n't be less than $0.04 (f'_c/f_y)$. **15**

In **1983, Hagberg** introduced a mathematical model that allows for the determination of the capacity of various type of reinforcement, including both secondary and main reinforcement. This model is applicable across a practical range of av/d ratios, ranging from 0.15 to 1.5, and can be used for any combination of horizontal and vertical loads. Hagberg demonstrated the suitability of the truss analogy within this framework. The following formulas were proposed by Hagberg:

$$\left(1 - \frac{2 \cdot f_c' \cdot b \cdot d}{F_s}\right) \cdot \tan^2 \beta + \left(\frac{2 \cdot f_c' \cdot b \cdot a}{F_s}\right) \cdot \tan \beta + 1 = 0 \quad \text{..........} \mathbf{16}$$

Where: $F_s = F_{s1} + F_{s2}$, $F_{s1} = A_s \cdot f_y$ and $F_{s2} = A_h \cdot f_{vy}$

Ah and As are the secondary and main reinforcement respectively in mm2.

Fvy and Fy are the yield strength of secondary and main reinforcements respectively in MPa.

$$d = \frac{d_1 \cdot Fs1_{\times d2.Fs2}}{F_s},$$

Where d1 and d2 are main reinforcement's separation and the secondary reinforcement's Fs center of gravity, respectively, mm.

β, the inclination compression strut with the vertical.

f'_c the concrete cylinder strength, MPa.

(**Siao, 1994**) [14], a novel methodology has been created to ascertain the shear resistance of deep beam, corbels, and shear wall with limited height to length ratio, when subjected to top-loading. The structural system comprises of three constituent parts, each of which features a compression strut. In order to effectively convey shear force to the support, a strut-and-tie mechanism is employed. The ultimate shear capacity can be calculated utilizing the enhanced strut-and-tie configuration illustrated in Figure 3 in the following manner:

$$V_u = 1.8 f_t \, b \, d \quad \text{......} \mathbf{17}$$

Where: f_t, is the steel yield strength.

(f_t), the allowed tensile strength of tension tie in refined compression strut, determined by equation (17a) before crack and by equation (17b) for crack concrete, where steel reinforcement shall with stand all stress.

$$f_t = 7 \cdot \sqrt{f_c'} \cdot [1 + n(\rho_h \cdot \sin^2\theta + \rho_v \cdot \cos^2\theta)] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ 17a$$

$$f_t = (\rho_h \cdot \sin^2\theta + \rho_v \cdot \cos^2\theta) \cdot f_y \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ \mathbf{17b}$$

(n), the ratio of steel and concrete's moduli of elasticity, ($n = Es / Ec$)

(ρ_v **and** ρ_h), steel reinforcement ratio of vertical and horizontal bar, respectively.

(θ), the inclination of compression strut to tension tie.

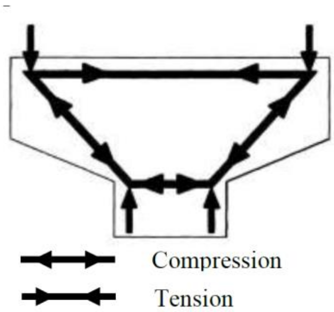

Fig. (3) - Refined Strut and Tie Model [14]]

(**Hwang et al, 2000**) [14], a modified strut-and-tie model was propose by the author to predict the shear capacity of corbels. The methodology under consideration was evaluated against a total of 178 samples as reported in the existing literature. The corbels under examination exhibit a range of factors, such as av/d ratios, diverse strength classifications, and horizontal reinforcement. Upon analyzing the selected test data, it was determined that the forecasts generated by the ACI empirical equations were overly cautious. The aforementioned conservatism was observed to be more pronounced in corbels possessing low aspect ratios (i.e., the ratio of height to length) or those constructed using high-strength concrete. It has been established that the utilization of vertical stirrups does not contribute to the enhancement of shear strength in a corbel with av/d ratio less than 1.

Hwang et al. [14] proposed a model that demonstrated the effectiveness of web reinforcement in corbels. The web reinforcement serves two purposes: firstly, it creates tension links and facilitates the transfer of shear, and secondly, it helps regulate crack widths and slows down the softening process of fractured concrete. Figure 4-a provides a detailed representation of the corbel studied in the research.

To cover a wide range of practical scenarios, the following parameters were selected: (h fyh) values ranged from 0 to 8 MPa for regular strength concrete and high-strength concrete, while (av/d) values ranged from 1/4 to 1. The compressive strength of regular strength concrete was determined to be 30 MPa, while high-strength concrete had a compressive strength of 70 MPa. Figures 2.4-b and 2.4-c display the predicted shear strengths of the corbels using the softened strut-and-tie model. It was observed that the upper limit of concrete strength is defined by (h fyh), and this conservative approach is particularly evident in structures with high aspect ratios (av/d) or those constructed using high-strength concrete. The study found that the use of vertical stirrups does not significantly increase the shear strength of a corbel with an av/d ratio less than 1.

In another study by **Aziz 2001** [15], the impact of crushed stone on the shear strength of reinforced concrete corbels was examined. It was discovered that concrete with crushed stones exhibited higher compressive strength, tensile strength, and shear stress values compared to concrete with natural gravel, under similar conditions of percentage, workability, curing, and testing. The compressive strength of concrete increased proportionally with an increase in shear stress and the amount of longitudinal and shear reinforcement. However, the compressive strength decreased as the ratio of shear span to depth (av/d) increased. The strength of concrete had a direct relationship with this limit, meaning that an increase in concrete strength corresponded to an increase in this limit.

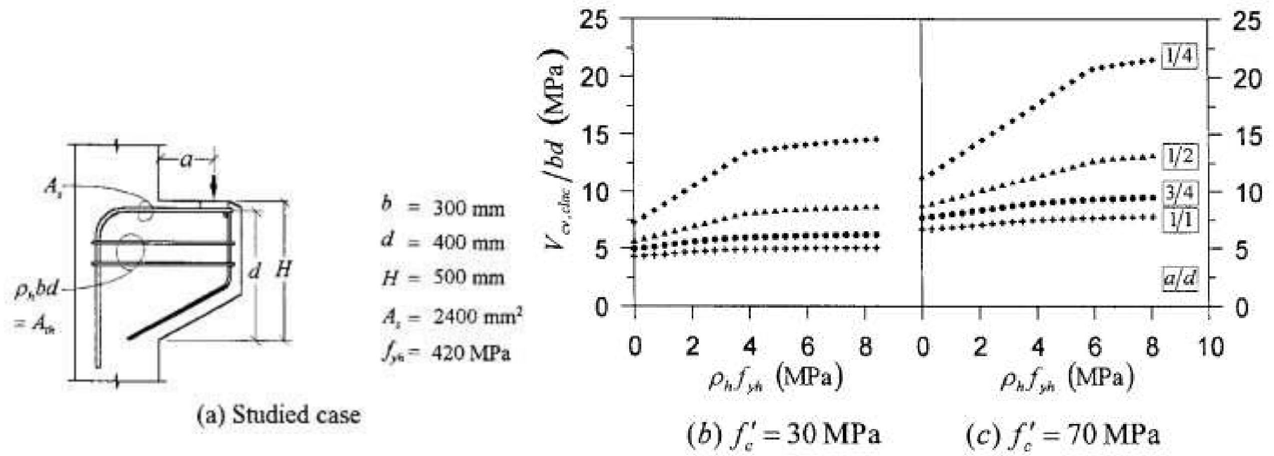

Fig. (4) – Verification of Shear Strength Calculation Considering the Influence of Horizontal Hoop Amount (f) and Shear Span-to-Depth Ratio (a/d)

The equation for shear was formulated based on the results of this test and additional data obtained from literature, which included the collapse of 168 reinforced concrete corbel due to shear.

$$v_u = 2.38 \times \left(\frac{f_c' \times k/d \times (\rho_w + \rho_h)}{a/d}\right)^{0.175} \qquad \text{...... } \mathbf{18}$$

□□ □ the ultimate shear stresses of reinforced concrete corbel in MPa
□***fc*′**□ compressional strength of concrete in MPa
□***k,d***□ the section's characteristics, k= 150 mm
□***a /d***□ the shear span / depth ratio
□□□w **,**□□h □ the longitudinal and shear reinforcement ratio.

Russo et al. (2006) conducted a study with the objective of tackling the difficulties related to the prediction of corbels' shear strength. The aim was to formulate a singular and exact mathematical representation that would obviate the necessity for protracted computational methodologies. The equation denoted as (20) illustrates the mathematical expression employed for the computation of the shear strength of a corbel. This formula was derived through a thorough examination of 243 experimental data points obtained from diverse literature sources.

$$v_u = 0.5 \cdot \left(k \cdot \chi \cdot f_c' \cdot \cos\theta + 0.65 \cdot \rho_k \cdot f_{\text{yम}} \cdot \cot\theta\right) \qquad \text{...... } \mathbf{19}$$

Where:(k), the derivation is based on the classical bending theory applied to reinforced concrete beams that are reinforced solely with tensile reinforcement.

$$k = \sqrt{(n \cdot \rho_f)^2 + (2 \cdot n \cdot \rho_f)} - (n \cdot \rho_f) \qquad \text{...... } \mathbf{20\ a}$$

(n), the ratio of the elastic moduli of steel and concrete, $(n = E_s/E_c)$

(ρ_f), the flexural reinforcement ratio

$$\rho_f = \frac{A_s - A_n}{b \cdot d} \quad \text{…… 20 b}$$

$$A_n = \frac{N_w}{f_{ys}} \quad \text{…… 20 c}$$

(f_{y5}), the yield strength of the main reinforcement.

(f_{jp}), the yield strength of the stirrups.

(ρ_h), the stirrup ratio at column-corbel interface.

$$\rho_f = \frac{A_h}{b \cdot d} \quad \text{…… 20 d}$$

(θ), the angle between the compressive concrete strut and the vertical directions, and is provided by equation (20-e).

$$\theta = 2 \cdot \arctan\left(\frac{-1 + \sqrt{\left(\frac{a}{d}\right)^2 + 0.22\left(1 - \frac{k^2}{4}\right)}}{\frac{a}{d} - \frac{k}{2}}\right) \quad \text{…… 20 e}$$

(χ), provid by equation (20-f) with$(10 \le f_c' \le 105, MPa)$.

$$\chi = \left[0.74\left(\frac{f_c'}{105}\right)^3 - 1.28\left(\frac{f_c'}{105}\right)^2 + 0.22\left(\frac{f_c'}{105}\right) + 0.87\right] \quad \text{…… 20 h}$$

The author has exclude corbel that exhibit a flexural reinforcement quantity, represented as "ρf," that falls below the minimum threshold stipulated by the ACI-318-02 standard, namely "ρf min = 1.4/fys." Figure (5) illustrates the reinforced concrete corbel utilized in the study.

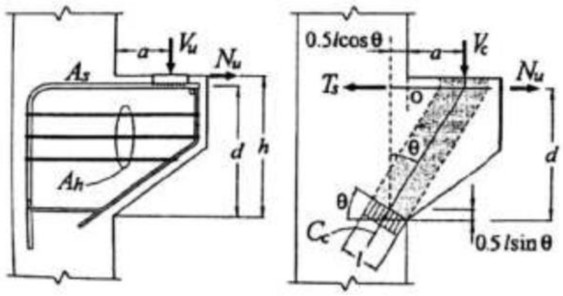

Fig. (5) - (a) RC Corbel Geometry and (b) Strut-and-Tie Model with Corbel Forces [17]

The design formula proposed was determined to be sufficiently conservative and reliable. It consistently resulted in a nearly constant safety factor, where the experimental shear strength (v Experimental) was in agreement with the calculated shear strength (v Calculated).

In their work, **He et al. 2012**[19] focused on theoretical models and explicit equations to enhance the understanding of shear behavior in concrete structures, particularly deep beams and corbels. They identified two primary mechanisms for shear transfer in deep beams: the direct strut mechanism and the truss mechanism. The direct strut mechanism involves the immediate transfer of load from the load point to the support, while the truss mechanism is the main method of shear resistance in slender beams.

The calculation of shear resistance for each mechanism depends on the aspect ratio of the structural element, specifically the ratio of span to depth. In structural members, the truss mechanism dominates when the shear span-to-depth ratio is two or greater, whereas the direct strut mechanism is more significant when the ratio is one or less. For members with axial-to-bending stiffness ratios between 1 and 2, a hybrid approach is observed, utilizing both mechanisms for load transfer.

The methodology employed by the authors used the maximum strength criterion to

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determine the load proportion transferred through each mechanism. This study established similarities in geometric attributes, loading configuration, and failure modes between deep beams and double corbels. This allowed for the use of concrete corbels as a model for strength analysis. The authors proposed an upper limit for the shear capacity of concrete corbels based on maximizing strength.

Overall, He et al. explored the shear-resisting mechanisms in deep beams and corbels, considering their geometric attributes, load distribution, and strength capacity. The findings from this study contribute to a better understanding of shear behavior in concrete structures.

$$\frac{1}{V_u} = \frac{1}{V_1^*} + \frac{1}{V_2^*} \qquad \ldots\ldots\ \mathbf{21}$$

$$V_u = \frac{k \cdot \tan\theta}{\left[1 - \frac{(\gamma_{ht} \cdot \sin^2\theta)}{2}\right]} \cdot v \cdot f_c' \cdot b \cdot d \qquad \ldots\ldots\ \mathbf{21\ a}$$

$$V_1^* = \frac{\tan\theta}{\left[1 - \frac{(\gamma_{ht} \cdot \sin^2\theta)}{2}\right.} \cdot v \cdot f_c' \cdot b \cdot d \qquad \ldots\ldots\ \mathbf{21\ b}$$

$$V_2^* = \frac{2n \cdot (1 - 0.6 \cdot \gamma_{ht}) \tan\theta}{\left[1 - \frac{(\gamma_{ht} \cdot \sin^2\theta)}{2}\right]^2} \cdot \frac{(v \cdot f_c')^2 \cdot b \cdot d}{f_y} \qquad \ldots\ldots\ \mathbf{21\ c}$$

Where:

$\boldsymbol{V_u}$ the total ultimate shear capacity

$\boldsymbol{V_1}^*$ ***and*** $\boldsymbol{V_2}^*$, terms associate with the concrete and steel contributions to shear resistances

$\gamma ht = \frac{\left(2 \cdot \frac{z}{a} - 1\right)}{3}$ For (0 ≤ γ ht <1)

Figure (6) depicts the load exerted upon a reinforced concrete corbel and the potential load pathways as per the Strut-and-Tie Method (STM).

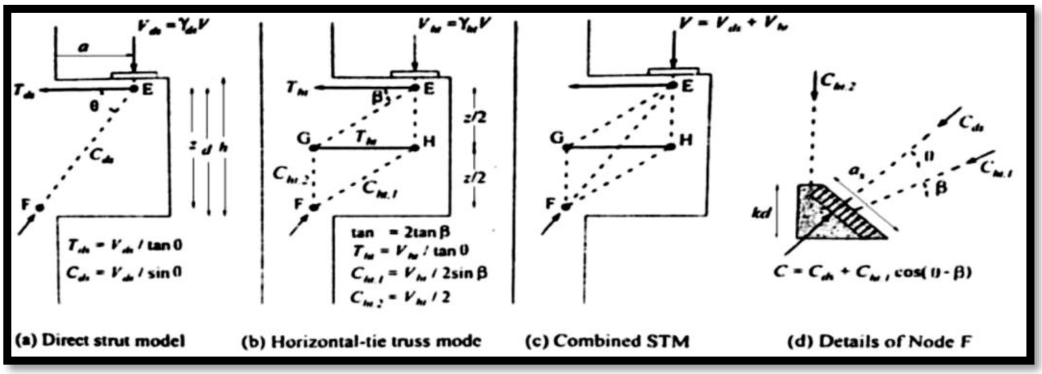

Fig. (6) – Strut-and-Tie Method for Concrete Corbel [19]

Summary and Conclusions:

After conducting a thorough review of experimental research on corbels, it was determined that there is a significant lack of studies examining corbels that have been subjected to repeated loading. Thus, additional inquiries are necessary to examine the effectiveness of strengthened corbels subjected to similar loading circumstances, which is the primary objective of the current research. The importance of this investigation stems from the fact that a significant proportion of corbels in practical scenarios are exposed to loading conditions of this type, which include dynamic loads from vehicular traffic on bridge girders supported by corbels, as well as corbels that provide support for cranes in warehouses, among other examples.

In conclusion, the existing literature suggests the following findings:

1. The maximum achievable strength (Vu) of a corbel is influenced by several factors, including its width (b), effective depth (d), reinforcement ratio (ρ), concrete strength (f'), and the ratio of shear span (av) to effective depth (d). The findings of these examinations indicate a direct relationship between the strength of corbels and the amount of longitudinal and shear reinforcement, as well as the compressive strength of the concrete. Conversely, the strength of corbels shows an inverse correlation with the shear span-to-depth ratio (av/d). Such loading conditions are commonly encountered in various applications, such as the dynamic loads exerted by vehicles on bridge girders supported by corbels or the support provided by corbels to cranes in warehouses, among other scenarios.
2. The stirrups, which function as horizontal reinforcement in corbels, demonstrate similar effectiveness to the primary tension reinforcement in withstanding vertical loads. The efficacy of stirrups in situations that entail concurrent loading is restricted, thereby rendering any reinforcement offered by them as additional support. Therefore, it is essential to guarantee the provision of the minimum amount of stirrup.
3. According to several researchers, corbels with a low shear span-to-depth ratio (av/d) are not suitable for analysis using the truss analogy. Somerville (1972) specifically restricted the application of the truss analogy to av/d ratios equal to or greater than 0.6.
4. The use of alternative reinforcement options, such as steel or polypropylene fibers, or plastic meshes, instead of traditional secondary reinforcement, has been observed to enhance the characteristics of concrete corbels.
5. Based on the analysis conducted, it can be concluded that the utilization of vertical stirrups does not result in an improvement of shear strength in a corbel where the ratio of the effective shear span to the effective depth is less than or equal to one.
6. It is widely agreed among scholars that the provisions for corbels as stipulated in the ACI-318 code were excessively conservative in their application to high-strength concrete.
7. The ACI-318 code mandates the inclusion of brackets and corbels, with a minimum volume fraction requirement for main bars. However, further research is needed to determine the optimal maximum limit for primary bars that can be effectively utilized in corbels.
8. The applicability of the finite element method in evaluating normal and high-strength concrete corbels is widely recognized. This approach holds promise for accurately predicting the behavior of reinforced concrete structures in a more comprehensive manner. The use of this methodology is deemed feasible for analysis and design purposes, particularly considering the rising costs associated with experimental testing and the declining expenses associated with computational techniques.
9. The implementation of the strut-and-tie model represents a feasible and effective methodology for the design of reinforced concrete corbels, given its capacity to offer reasonably precise predictions of their ultimate load-carrying capacity.

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