About one Problem of Analysis and Control in Systems with Distributed Parameters on the Example of Oil Fields

Suvonov Olim Omonovich Associate Professor of Navoi State Pedagogical Institute, Candidate of Technical Sciences

ABSTRACT

The paper considers the solution of the problem of optimal control of systems with distributed parameters, described by partial differential equations of elliptic type, as applied to oil fields. A qualitative analysis of the states of the system has been carried out by solving a boundary value problem in the class of generalized functions.

Keywords: System, distributed parameter, differential equation, elliptic, solution, generalized function, Helbert, Banach, normed space, smoothness of solutions, summable, oil field.

The development of oil and gas fields can be considered as a dynamic system in which, under the influence of input variables (technological modes of operation of wells), the controlled variables (oil and gas reserves in productive strata, reservoir pressure) change.

Oil and gas fields, as objects of modeling and optimization, are characterized by a significant number of interconnected hydrodynamic, technological and economic parameters that change in the process of system control [1,2].

Recently, a number of problems have arisen for managing unstable processes in such spatially distributed objects as geofiltration. In this regard, it was necessary to develop an appropriate theory that takes into account certain specific conditions for the physical and technical feasibility of control laws, giving effective methods in the rational development of oil and gas fields [3,4].

The paper will show the possibility of applying methods for solving some problems of optimal control of systems described by partial differential equations. Let for a given control $g(x)$ state of the system $u(x)$ can be found from the solution of a differential equation with partial derivatives of an elliptic type, as applied to oil fields.

Let for a given control $g(x)$ state of the system $u(x)$ can be found from the solution of a partial differential equation of elliptic type

$$L_u = \sum_{i,j=1}^{n} \frac{\partial}{\partial x_i} a_{ij}(x) \frac{\partial u}{\partial x_j}(x) - a(x)u(x), \qquad x \in \Omega \qquad (1)$$

under conditions

$$\begin{cases} u(x)=0, & x\in S_0, \\ \dfrac{\partial u}{\partial N}(x)=g(x), & x\in S. \end{cases} \qquad (2)$$

The state $P(x)$ (output parameters of the object) is defined as

$$P(x)=u(x), \qquad x\in S, \qquad (3)$$

where Ω - bounded enclosed area n - dimensional space R^n; $\delta\Omega = S_0 \cup S, S_0 \cap S = \theta$ - region border Ω;

$\dfrac{\partial u}{\partial N}(x)=\sum_{i,j=1}^{n} a_{ij}(x)u_{xj}(x)\cos(\vec{n},x_i)$ - function derivative $u(x)$ along the co normal to the surface S , $\vec{n}$ - internal normal vector to S.

In view of the fact that it is not possible to prove the existence of a solution to the formulated optimal control problems in the space of classical solutions of the differential equation (1), we will consider these problems in the space of generalized solutions.

According to [5-9], the function $u(x)\in W$ is called a generalized solution of differential equation (1) if for any function $\eta(x)\in W$ the identity

$$\pi(u,\eta)=\int_{\Omega}\left[\sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial u}{\partial x_i}\frac{\partial \eta}{\partial x_j}(x)+a(x)u(x)\eta(x)\right]dx=-\int_{S} g\eta dx. \qquad (4)$$

Here and below, the following notation is used

$L_g(\Omega),[L_g(S)]$ - the Banach space consisting of all measurable functions that are Lebesgue summable in the domain $\Omega\,[S]$ with a degree g , with the norm

$$\|u\|_{L_g(\Omega),[L_g(S)]}=\left[\int_{\Omega[S]}|u(x)|^g\,dx\right]^{\frac{1}{g}},$$

$$\|u\|_{L_g(\Omega),[L_g(S)]}=vrau\max_{x\in\Omega(S)}|u(x)|;$$

$W_2^i(\Omega), \quad i=1,2,$ hilbert space consisting of all elements $L_2(\Omega)$, having generalized derivatives up to the order i inclusive, square summable over Ω with norm [5]

$$\|u\|_W=[\int_{\Omega}[u^2(x)+\sum_{i=1}^{n}(\frac{\partial u}{\partial x_i})^2(x)]dx]^{\frac{1}{2}},$$

$$\|u\|_{W_2^2}=[\int_{\Omega}\{u^2(x)+\sum_{i=1}^{n}(\frac{\partial u}{\partial x_i})^2(x)+\sum_{i,j=1}^{n}(\frac{\partial^2 u}{\partial x_i\partial x_j})^2(x)+\}dx]^{\frac{1}{2}};$$

W - set of functions that belong $W_2^i(\Omega)$, equal to zero on S_0; W_A - subset W, consisting of functions equal to zero on a measurable set $A\in S$. As is known [8,9], with sufficient smoothness of the boundary $\delta(\Omega)$, W and W_A - closed subspaces of space $W_2^i(\Omega)$. $Z(x,y)$ - the space of continuous linear operators from the topological space X into the topological space Y.

Region Ω belong to class $C^{2+\alpha}$, $0<\alpha<1.$ (5)

Regarding the parameters of the object, we will assume that the conditions (5)

$$a_{ij}(x), \frac{\partial a_{ij}}{\partial x_i}(x), \quad i, j = 1,2,\dots,n \text{ - continuous in } \overline{\Omega} \quad (6)$$

functions, $a \in L_\infty(\Omega)$. Exist $\upsilon, \mu > 0$ such that for any n - dimensional vector $(\xi_1, \xi_2, \dots, \xi_i, \dots, \xi_n)$ the inequality

$$\nu \sum_{i=1}^{n} \xi_i^2 \le \sum_{i,j=1}^{n} a_{ij}(x)\xi_i \xi_j \le \mu \sum_{i=1}^{n} \xi_i^2, \quad x \in \overline{\Omega}. \quad (7)$$

$$Du + Bg = 0. \quad (13)$$

Let be C - operator that maps functions $u \in W$ operator that maps functions S

$$Cu = P. \quad (14)$$

As is known [9, 10], under conditions (5) the trace $P(x)$ to the border S any function $u \in W$ is an element of the complete normed space $H(S) = W_2^{1/2}(S)$, owned $L_2(S)$. There is an inequality $\|u\|_H \le a\|u\|_W$. Therefore, the range of the operator C belongs $H(S)$ and $C \in Z(W, H(S))$.

Using the results of the continuation theory [10] and the averaging of functions [9], one can verify that the space $H(S)$ satisfies the assumptions of the system of equations adjoint to (13), (14) of the following form

$$\begin{cases} D^* y + C^* v = 0 \\ C^* y = \psi, \end{cases} \quad (15)$$

where D^*, C^*, B^* - operators conjugate D, C, B.

For a given $v \in L_2(S)$ function $y \in W$, satisfying (15) can be found from the solution of the equation

$$\pi(y,\eta) = \int_\Omega [\sum_{i,j=1}^{n} a_{ij} \frac{\partial y}{\partial x_i} \frac{\partial \eta}{\partial x_j} + ay\eta] dx = -\int_S v\eta dx, \quad \forall_\eta \in W.$$

Operator B^* assigns functions $y \in W$ her trace. To the border S.

<u>Remark 1,</u>

The solution of identity (4) can be interpreted as follows. By virtue of the theorem from [9], under conditions (5) - (9), the function $y \in W$, satisfying (4) belongs to the space $W_2^2(\Omega')$ for any strictly internal region $\Omega' \subset \Omega$, which means that for almost everyone $x \in \Omega$ this function satisfies (1)[8]. With this in mind, and using arguments completely analogous to those given in [9], it is easy to show that for any $\eta \in W$

$$\lim_{k\to\infty} [\int_{S_k} \frac{\partial u}{\partial N_k} \eta dx] = \int_S g\eta dx,$$

where Ω_k - arbitrary increasing sequence of regions having a sufficiently smooth boundary $\delta\Omega_k = S_k \cup S_0$, contained in Ω and striving for Ω.

The following theorem on the properties of equations (13), (14) is valid.

<u>Theorem 1.</u>

Under conditions (5) - (9) for any function $g \in L_2(S)$ and measurable space $A \subset S$ there is a point $(g_A, u_A, P_A) \in L_2(S) \times W \times H(S)$, satisfying equations (13), (14) and conditions

$$\begin{cases} P_A(x) = 0, \quad x \in A, \\ g_A(x) = g(x), \quad x \in S \setminus A. \end{cases}$$

(16)

function $u_A \in W_A$ can be found from the solution of the equation

$$\pi(u_A,\eta)=\int_{\Omega}[\sum_{i,j=1}^{n} a_{ij}\frac{\partial u_A}{\partial x_i}\frac{\partial \eta}{\partial x_j}+au_A\eta]dx=-\int_{S\setminus A} g\eta dx, \quad \forall_\eta \in W_A.$$
(17)

Wherein $g_A=\begin{cases} g(x), x\in S\setminus A, & P_A(x)=u_A(x), x\in S. \\ \frac{\partial u_A}{\partial N}(x), x\in A, \end{cases}$

There is only one function $u_A \in W_A$, satisfying (17) functions g_A and P_A belong respectively to the space $L_2(S)$ и $H(S)$.

Proof:

Let us show that the function $u_A \in W_A$, satisfying identity (17) exists and is unique.

Due to conditions (7) - (8) $\pi(u,u)\geq \nu\int_{\Omega}\sum_{i=1}^{2}(\frac{\partial u}{\partial x_i})^2 dx$.

At the same time, for any $u\in W_A$ the inequality [10]

$$\int_{\Omega} u^2 dx \leq a\int_{\Omega}\sum_{i=1}^{n}(\frac{\partial u}{\partial x_i})^2 dx.$$

Therefore

$\pi(u,u)\geq c\|u\|_W^2$, where

$$c=\min(\frac{\nu}{2},\frac{\nu}{2a})>0 . \qquad (18)$$

Because $\pi(u,\eta)$ - continuous bilinear form in space W_A, a $(g,\eta)_{L_{2(S\setminus A)}}$ - continuous linear functional in W_A. (see (11), (12)), then under conditions (18) there is a unique element $u_A \in W_A$, satisfying for anyone $\eta\in W_A$ identity (17), (the Vishink and Lax-Milgrange lemma [6]).

Let us show that $u_A \in W$ satisfies identity (4). According to the theorem from [9], under conditions (5) - (9), $u_A \in W_2^2(\Omega')$ for any subset $\Omega' \subset \Omega$ c fairly smooth border $\delta\Omega$, with $\delta\Omega$ general part $A' \in A$, and consequently, $\frac{\partial u_м}{\partial N}\in L_2(A)$ [10] and u_A suits almost everyone $x\in\Omega$ equation (1) (see remarks 3.1). Therefore, for any $\gamma\in W_{S\setminus A}$ the identity [9]

$$\pi(u,\gamma)=\int_{\Omega}[-\sum_{i,j=1}^{n}\frac{\partial}{\partial x_i}a_{ij}\frac{\partial u}{\partial x_j}+au]\gamma dx=-\int_{A}\frac{\partial u_A}{\partial N}\gamma dx$$

. (19)

Since under conditions (5) the subspace of functions $\eta\in W$, represented in the form $\eta=\gamma+\xi$, where $\gamma\in W_{S\setminus A}$, $\xi\in W_A$, tight in W и $u_A \in W$ satisfies (17), (19), then for any $\eta\in W$ the identity

$$\pi(u_A,\eta)=\pi(u_A,\gamma)+\pi(u_A,\xi)=-\int_{A}\frac{\partial u_A}{\partial N}\gamma dx--\int_{S\setminus A} g$$

,

where $g_A=\begin{cases}\frac{\partial u}{\partial N}(x), & x\in A, \\ g(x), & x\in S\setminus A.\end{cases}$

This identity proves that the point (g_A,u_A,P_A) satisfies (13), (14) and conditions (16).

Theorem 2.

If point (g,u,P) satisfies equations (13), (14), then

a) $\|u\|_W^2 \leq a_1/(g,P)L_{2(S)}/$,

b) $/(g,\eta)_{L_{2(S)}}/\leq a_2\|u\|_W\|\eta\|_H$ for anyone $\eta\in H(S)$.

Here and further $g,v\in L_2(S)$, $u,y\in W, P$, $\psi\in H(S)$.

Proof.

If point (g,u,P) satisfies equations (13), (14), then $\pi(u,u)=-(g,P)_{L_2(S)}$. In view of this and inequality (18),

$$c\|u\|_W^2 \leq /\pi(u,u)/=/(g,P)_{L_2(S)}/ .$$

Since any function $\eta \in H(S)$ can be extended into the region Ω, so $\eta \in W$ and $\|\eta\|_W \le a_3 \|\eta\|_H$, where is the constant a_3 does not depend on η [8], then using (11), we obtain the inequality

$$/(g,\eta)_{L_2(S)}/ = /\pi(u,\eta)/ \le a\|u\|_W \|\eta\|_W \le a_2 \|u\|_W \|\eta\|_H$$

,

which completes the proof of theorem 2.

Property 3.

Let the point (g,u,P) satisfies equations (13), (14), then

a) if $g(x) \le 0, x \in S \setminus A, \quad P(x) \ge 0, x \in A$, then $P(x) \ge 0$ for almost everyone $x \in S$,

б) if $g(x) \le 0, x \in S \setminus A, \quad P(x) = 0, x \in A$, then $g(x) \ge 0$ for almost everyone $x \in S$,

Here A measurable subset belonging to S.

Proof.

a) we will carry out the proof using the method proposed in [9] for the same theorems.

Let the point (g,u,P) satisfies equations (13), (14) and $\underset{x\in S\setminus A}{vrai\max}\, g(x) \le 0, \underset{x\in A}{vrai\min}\, P(x) \ge 0.$ Then for any $\gamma \in W_A$ the identity $\pi(u,\gamma) = -\int\limits_{S\setminus A} g\gamma dx$.

So if $\underset{x\in S\setminus A}{vrai\min}\, \gamma(x) \ge 0,$ then $\pi(v,\gamma) \le 0,$ где

$$v(x) = -u(x),\ x \in \Omega. \tag{20}$$

Let's pretend that

$$\underset{x\in\Omega}{vrai\max}\, v(x) = M > 0 \tag{21}$$

Let be $v^0(x) = \max(v(x),0)$. Because $v(x) = -u(x) \le 0$ for almost everyone $x \in A$ то $v^0 \in W_A$, it's clear that $\underset{x\in S\setminus A}{vrai\max}\, v^0(x) \ge 0$.

Therefore, in (20) one can put $\gamma = v^\circ$. Then, using (18), (20) , we obtain the inequality

$$\|v\|^2_{W_2^{\cdot}(\Omega_\circ)} \le \int\limits_{\Omega_\circ} [\sum_{i,j=1}^n a_{ij} \frac{\partial v}{\partial x_i}\frac{\partial v}{\partial x_j} + av^2]dx = \int\limits_{\Omega} [\sum_{i,j=1}^n a_{ij} \frac{\partial v}{\partial x_i}\frac{\partial v^0}{\partial x_j} + avv^0]dx \le 0$$

,

where $\Omega_0 = \{x \in \Omega : v(x) > 0\}$, which proves that the measure is a set Ω_0 is equal to zero, which contradicts assumption (21). In view of this $\underset{x\in\Omega}{vrai\max}\, v \le 0$, which means $u(x) = -v(x) \ge 0$, for almost everyone $x \in \Omega$. Since under conditions (5) the function $u \in W$ the equivalent of a continuous Ω function, which is uniquely extended by continuity for almost all points of the boundary [10], then $P(x) = u(x) \ge 0$ for almost everyone $x \in S$.

b) let point (g,u,P) satisfies (13), (14) and conditions $P(x) = 0, \quad x \in A, \quad g(x) \le 0, x \in S \setminus A$. As noted in [10], under conditions (5) there is a sequence of functions $u^k \in C^{2+\alpha}(\bar{\Omega})$, converging to $u(x)$ according to the norm of space W and satisfying the conditions

$$P^k(x) = u^k(x) \ge 0, \quad x \in S \setminus A,$$

$$P^k(x) = u^k(x) = 0, \quad x \in A \cup S_0,$$

where $C^{i+\alpha}(\bar{\Omega}), \quad i = 0,1,2, \quad 0 < \alpha < 1$- space of Hölder continuous functions with exponent α in area $\bar{\Omega} = \Omega \cup \delta\Omega$ together with derivatives up to i order inclusive. Take sequences of functions

$$a_{ij}^k \in C^{1+\alpha}(\bar{\Omega}), i,j = 1,2,\ldots,n;\ a^k \in C^\alpha(\bar{\Omega}),\ k = 1,2,\ldots \tag{22}$$

satisfying conditions (6) - (9) and converging to $a_{ij}(x), a(x)$ according to the norm $L_\infty(\Omega)$.

Let's find functions $y^k(x)$ from the solution of the equation

$$\sum_{i,j=1}^n \frac{\partial}{\partial x_i} a_{ij}^k(x) \frac{\partial y^k}{\partial x_j}(x) - a^k(x) y^k(x) = 0,\ x \in \Omega, \tag{23}$$

under boundary conditions

$$y^k(x) = P^k(x).$$

(24)

According to the theorem from [9], under the conditions formulated above, there is a unique function $y^k(x) \in C^{2+\alpha}(\bar{\Omega})$, satisfying (23), (24). Compute $g^k(x) = \frac{\partial y^k}{\partial N}(x), \quad x \in S$. Note that $g^k(x) \in L_2(S)$ and for any $\eta \in W$ the identity

$$\pi^k(y^k,\eta) = \int_\Omega [\sum_{i,j=1}^n a_{ij}^k \frac{\partial y^k}{\partial x_i}\frac{\partial \eta}{\partial x_j} + a^k y^k \eta]\, dx = -\int_S g^k \eta dx$$

.

By virtue of the theorem from [9], under conditions (22)

$$\lim_{k\to\infty} \left\| y^k - u \right\|_W = 0.$$

(25)

Having carried out calculations that are completely similar to those given in [9], we can show that under conditions (22), (25)

$$\lim_{k\to\infty} / \pi(u,\eta) - \pi^k(y^k,\eta)/ = 0, \quad \forall_\eta \in W .$$

Because

$$/\pi(u,\eta) - \pi^k(y^k,\eta)/ = /(g^k - g,\eta)_{L_2(S)}/,$$

then for any $\eta \in A(S)$

$$\lim_{k\to\infty} / (g^k - g,\eta)_{L_2(S)}/ = 0.$$

(26)

Due to the fact that $\min_{x\in\Omega} y^k(x) \geq \min_{x\in S} P^k(x) = 0$ [9], and $P^k(x) = 0, x \in A,$ then $y^k(x)$ reaches a minimum at any point $x \in A$.

Taking into account that $g^k(x) = \frac{\partial y^k}{\partial N}(x)$ - derivative along the inward direction to the boundary S, we have

$$g^k(x) \geq 0, \quad x \in A.$$

(27)

Since the set of functions $\eta \in H(S)$, equal to zero on $S \setminus A$ tight in $L_2(A)$, then from (26), (27) we obtain $g^k(x) \geq 0$ for almost everyone $x \in A$ and the theorem is proven.

The above theoretical study on the analysis of system states in the process of optimal control of a deterministic control object can be successfully applied in a qualitative study of applied problems in the development of oil fields.

References

1. Suvonov O.O. Numerical algorithm of computational experiment of the applied optimal control problem in systems with distributed parameters. “Bulletin of TUIT: Management and Communication Technologies”Suvonov O.O. 2021, 2 (46) 1 UDK 62-50:276.681. 10 стр.
2. Suvonov O.O., Nazirova E.Sh. Mathematical modeling of the unctioning of a hydrodynamic system with distributed. Bulletin of TUIT: Management and Communication Technologies. 5 -23-2021 UDK 62-50:276.681. 10 стр.
3. Suvonov O.O., Kuchkarova S.S.Computational experiment of numerical study of hydrodynamic processes in interacting formations. Cite as: AIP Conference Proceedings 2365, 070017 (2021); https://doi.org/10.1063/5.0057574 Published Online: 16 July 2021. 7 стр.
4. Suvonov O.O. Mathematical model of control of a hydrodynamic object with distributed parameters (on the example of oil fields). Central Asian journal of mathematical theory and computer sciences

http://cajmtcs.centralasianstudies.org/index.php/CAJMTCS Volume: 03 Issue: 04 | Apr 2022 ISSN: 2660-5309.
5. Lyons J.-L. On inequalities in partial derivatives. - Success Math. Nauk, 1971, v. XXVI, no. 2.
6. Lyons J.-L. On optimal control of distributed systems. - Success Math. Nauk, 1973, v. XXVIII, no. 4.
7. Lyons J.-L. Optimal control of systems described by partial differential equations. M., Mir, 1972.
8. Lions Zh.L., Magenes E. Inhomogeneous boundary value problems and their applications. Mir Publishing House, Moscow 1971 386 p.
9. Ladyzhenskoya O.A., Uraltseva N.N. Linear and quasilinear equations of elliptic type. Publishing house "Science", 1973. 586 p.
10. Krivenkov Y.P. Sufficient optimality conditions for problems with second-order differential equations of elliptic type in the presence of phase constraints. Differential equations. T. XI, No. 1, 1975.