Mathematical Model of a Multiparameter Learning Process

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ABSTRACT

The issues of modeling the learning process as a control object with two or more parameters are considered. The multi-parameter model of the learning process is described in the form of ordinary differential equations. The results of computational experiments are illustrated in the form of graphs.

Keywords: learning process, computer model, mathematical model, differential equation, requirement, teacher, knowledge, strong, fragile, level, speed, increases.

Introduction

In the theoretical study and modeling of learning processes, as a multi-parameter object of social systems, a special place is occupied by the systematic approach of the science of cybernetics, based on the consideration of the didactic system "teacher - student" from the point of view of control theory, as well as methods of mathematical and simulation (computer) modeling. The essence of this approach is that the real learning process is replaced by an abstract model - some idealized object that behaves like the system being studied. Such a model can be a system of logical rules, differential equations, or a computer program that allows a series of computational experiments to be carried out for various parameter values, initial conditions, and external influences. By changing the initial data and the values of the model parameters, one can explore the ways of the system development, determine the given and predict the future state of the system.

There are known discrete and continuous models based on the automatic approach and the solution of differential equations [4, 7, 10]. In some cases, multi-agent modeling is used, in which each student is replaced by a software agent that functions independently of other agents [6]. There are also simulation models using Petri nets, genetic algorithms, matrix modeling [4-7].

The listed models do not take into account the elements of educational material learned by the student, they are not equal. Those elements of the educational material that are included in the student's activity turn into solid knowledge and are forgotten more slowly, and those that are not included are faster. In the process of learning, fragile knowledge gradually becomes strong. The study consists in creating a simulation model of the learning process that takes into account the difference in the speed of forgetting various elements of educational material and the transition of fragile knowledge into the category of solid knowledge. Let us assume that the computer simulation will more closely match the real learning process, given the following:

1) the strength of the assimilation of various elements of the educational material is not the same, therefore, all elements of the

educational material should be divided into several categories;

2) strong knowledge is forgotten much more slowly than weak knowledge;

3) Fragile knowledge, when used by students, gradually becomes strong.

Multiparametric model of the learning process.

The process of assimilation and memorization of transmitted information consists in establishing associative links between new and existing knowledge. As a result, acquired knowledge becomes more durable and is forgotten much more slowly. Repeated use of knowledge leads to the formation of appropriate skills and abilities in the student, which remain for a long time.

Denote by T the level of requirements set by the teacher and equal to the number Y_0 reported elements of educational material. Let be Y – total knowledge of the student, which includes knowledge of the first, second, third and fourth categories: $Y = Y_1 + Y_2 + Y_3 + Y_4$. При Y_1 – the most fragile knowledge of the first category with a high forgetting rate γ_1, a Y_4 – the strongest knowledge of the fourth category with low $\gamma_4 (\gamma_4 < \gamma_3 < \gamma_2 < \gamma_1)$. Absorption rates α_i characterize the speed of knowledge transfer $(i-1)$ - th category in knowledge i - th category. The proposed four-parametric learning model is expressed by differential equations:

$$\frac{dY_1}{dt} = k\alpha_1 (T - Y) Y^b - k\alpha_2 Y_1 - \gamma_1 Y_1;$$

$$\frac{dY_2}{dt} = k\alpha_2 Y_1 - k\alpha_3 Y_2 - \gamma_2 Y_2;$$

$$\frac{dY_3}{dt} = k\alpha_3 Y_2 - k\alpha_4 Y_3 - \gamma_3 Y_3;$$

$$Y = Y_1 + Y_2 + Y_3 + Y_4.$$

In the learning process $(k=1)$, the rate of increase in the student's fragile knowledge is proportional to: 1) the difference between the level of the teacher's requirements T and general level of knowledge Y; 2) the amount of knowledge already available Y to the extent b. The latter is explained by the fact that the availability of knowledge contributes to the establishment of new associative links and the memorization of new information. If the increase in the student's knowledge is significantly less than their total amount, then $b = 0$. When learning stops $(k = 0)$, Y decreases due to forgetting. Forgetting rate $\gamma = 1/\tau$, where τ – the time during which the amount of knowledge i - th portion decreases in $e = 2.72...$ times. The learning outcome is characterized by the total level of acquired knowledge $Y = Y_1 + Y_2 + Y_3 + Y_4$ и strength factor $\Pr = (Y_2/4 + Y_3/2 + Y_4)/Y$. If all the knowledge acquired during the training is fragile $(Y_1 = Y,\ Y_2 = Y_3 = Y_4 = 0)$, then the strength factor $\Pr = 0$. We must strive for a situation where all the acquired knowledge is solid $(Y_4 = Y,\ Y_1 = Y_2 = Y_3 = 0)$, then $\Pr = 1$. With a long study of one topic, the level of knowledge Y increases to T, then there is an increase in the share of solid knowledge Y_4, strength grows $\Pr$.

Computational experiment using learning model.

Let us analyze some case arising in the learning process.

1. The teacher conducts three lessons, the level of requirements T_i for i - th lesson set *(i=1,2,3)*. We analyze the learning process of a student using four parametric models. The graph shows that during training, the total amount of knowledge *Y* the student grows, part of the fragile knowledge becomes more solid (Fig. 1.1). During breaks and after training, the level of fragile knowledge Y_1 decreases rapidly, and sound knowledge Y_4 forgotten much more slowly.

2. The teacher conducts three lessons, the level of requirements $T(t)$ for *i*- th lesson grows according to the law $T_i = a_i (t_i - t_{i0}) + b_i, \quad i = 1, 2.$

3. Let us analyze the learning process using a two-parameter model. The two-parameter learning model is expressed by differential equations:

$$\frac{dY_1}{dt} = k\alpha_1 (T - Y)Y^b - k\alpha_2 Y_1 - \gamma_1 Y_1;$$

$$\frac{dY_2}{dt} = k\alpha_2 Y_1 - \gamma_2 Y_2;$$

$$Y = Y_1 + Y_2.$$

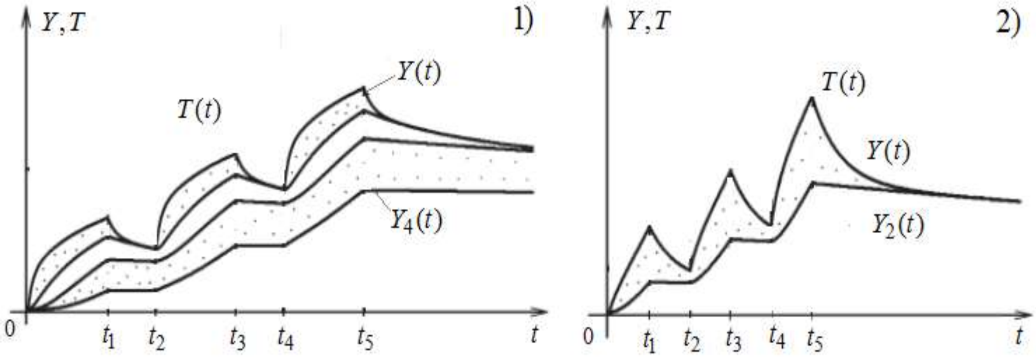

Fig. 1. Changing the level of requirements T teachers and the amount of knowledge Y_1 student in the learning process.

At each lesson, the teacher requires students (Fig 1.2):

1) possession of the material studied in previous lessons;

2) assimilation of new elements of educational material.

During training, fragile knowledge becomes solid and after training it is forgotten much more slowly.

3. The teacher must teach the student to decide N tasks of increasing complexity $\omega_i = i\Delta\omega$, which is considered equal to the amount of knowledge *Y*, required to solve *i*- th tasks. The teacher arranges the tasks in order of increasing complexity and sets them to the student at regular intervals. Δt. If the student does not decide *i*- task, then the teacher teaches him over time Δt, and then again offers the same or a similar problem of the same complexity ω_i. If the student's level of knowledge *Y* more ω_i, then the student is likely to solve the problem within Δt. Wherein *Y* will not increase, but part of the fragile knowledge will become solid. After that, the teacher gives him *(i+1)* - problem with a higher level of difficulty ω_{i+1}. If the student does not have enough knowledge, then with a greater probability he will not be able to solve the problem right away. teacher for time Δt explains the material, or the student studies according to the textbook; requirement level $T = \omega_i$, knowledge Y_1 and Y_1 are growing. The student then tries the problem again. Lessons duration T_3 □ Δt alternate with changes, duration T_{Π} □ Δt.

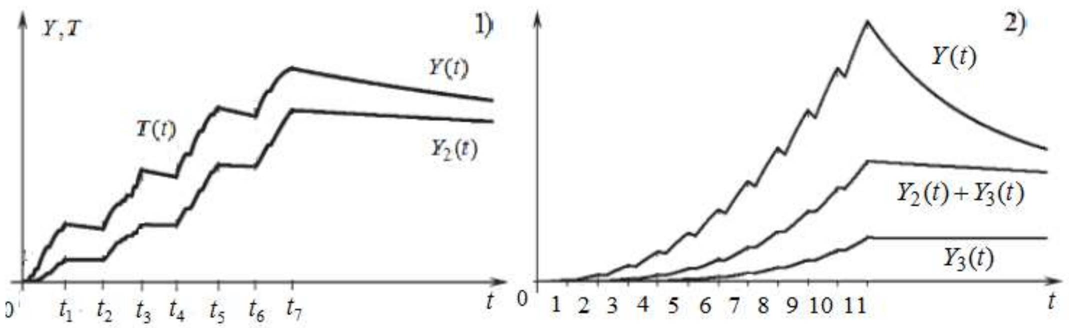

Fig. 2. Computer model of the learning process: 1-solving problems of increasing complexity; 2 - change in the amount of knowledge during schooling and after graduation.

In the program used, the solution of the problem is considered as a random process, the probability of which is calculated by the Roche formula:

$$p_i = 1/(1+\exp(-\lambda(Y(t)-\omega_i)).$$

At $Y=\omega$ decision probability *i*- th task is equal to p_i = 0,5. The results of simulation modeling of training in four lessons are presented in Fig. 2.1. step line $T(t)=\omega t$ shows how the complexity of the tasks being solved changes (the level of requirements); charts $Y(t)=Y_1+Y_2$ и $Y_2(t)$ characterize the growth dynamics of all and solid knowledge. The resulting curves are similar to the graph in Fig. 1.2 when requirements *T* during the lesson grow in proportion to time.

4. Schooling lasts 11 years. The academic year consists of 9 months of classes and 3 months of holidays. The level of demands placed by the teacher on the student in *i*- m class, given by the matrix $(T_1, T_2, ..., T_n)$. We will study the change in the student's knowledge during training and after its completion. A three-parameter learning model is used; typical simulation results are shown in Fig. 2.2. It can be seen how during the training the level of knowledge of the student grows, the amount of solid knowledge increases. Periodic Decreasing Graph *Y(t)* explained by forgetting during the holidays. After the end of training, fragile knowledge that the student rarely used is quickly forgotten, and solid knowledge is forgotten more slowly.

With the three-parameter learning model, questions of the formation of a system of empirical knowledge were used. At the same time, the entire set of factors studied at school was divided into three categories:

1) facts that can be established in everyday life;

2) facts established in a physical laboratory;

3) facts that are not established in the conditions of training and are studied speculatively.

After coordinating the computer model with the results of the pedagogical experiment, graphs were obtained that characterize the change in the level of knowledge of facts of various categories as the student studies at school.

To generalize the model, suppose that let *Y*– total knowledge of the student, Y_1 – the most fragile knowledge of the first category with a high forgetting rate γ_1, Y_2 – knowledge of the second category with a lower forgetting rate $\gamma_2, ...,$, a Y_n – the strongest knowledge *n*- th category with low $\gamma_n (\gamma_1 > \gamma_2 > ... > \gamma_n)$. Absorption rates α_i characterize the speed of knowledge transfer *(i–1)*- th category into more solid knowledge *i*- th category. Forgetting rate $\gamma = 1/\tau$, where τ – time, reducing knowledge by 2.72... times. Difficulty factor $S (0 \le S \le 1)$ allows you to take into account the subjective complexity of mastering *i*- th elements of educational material.

Learning is characterized by the amount of acquired knowledge *Y* and strength factor:

$$\Pr = \left(\frac{Y_2}{2^{n-2}} + ... + \frac{Y_{n-1}}{2} + Y_n\right) / Y.$$

When studying one topic, the level of knowledge first grows *Y*, then there is an increase in the share of solid knowledge Y_n and increased strength Pr. At any given time:

$$Y(t) = Y_1(t) + ... + Y_n(t).$$

During training:

$$\frac{dY_1}{dt} = r(1-S)(\alpha_1 F Y^b - \alpha_2 Y_1) - \gamma_1 Y_1;$$

$$\frac{dY_2}{dt} = r(1-S)(\alpha_1 Y_1 - \alpha_3 Y_1) - \gamma_2 Y_2$$

$$\cdots \quad \cdots \quad \cdots \quad \cdots$$

$$\frac{dY_n}{dt} = r(1-S)\alpha_n Y_{n-1} - \gamma_n Y_n.$$

Break time: $T = 0, \quad dY_1 / dt = -\gamma_1 Y_1,$ $dY_2 = -\gamma_2 Y_2, ..., dY_n / dt = -\gamma_n Y_n.$

The use of the proposed model allows us to analyze various situations encountered in pedagogical practice and take into account the influence of the complexity of the studied material and other factors on the learning outcome [9].

Conclusion

The proposed computer models complement qualitative reasoning, make them more objective, justified and can be used when conducting a pedagogical experiment is illegal or leads to negative results. By changing the sequence of studying various elements of educational material, the duration of classes, etc., one can find the optimal way of learning in a particular case.

One of the areas of using computer simulation of the learning process is associated with the creation of a training program that simulates the educational process at school and is intended for training students of pedagogical universities. It should allow changing the parameters of students, the duration of classes, the distribution of educational material and the strategy of the teacher's behavior. In the course of its work, a student playing the role of a teacher changes the speed of supply of educational information, quickly responds to students' questions, conducts tests, puts marks, trying to achieve the highest level of knowledge in a given time. After the end of "training", graphs are displayed on the screen showing the change in the "amount of knowledge of the students in the class", marks for "completed tests", etc. In addition, the training program analyzes the work of the "teacher" (student) and evaluates him.

References

1. Suvonov O.O., Jurakulov T.T. On one problem of mathematical modeling of learning processes as an object of management - Electronic journal of actual problems of modern science, education and training. june, 2020-III. ISSN 2181-9750 Urganch 56-67 pp.
2. Yodgorov G.R., Jurakulov T.T, Mathematical modeling of learning processes based on the theory of control - Cite as: AIP Conference Proceedings 2365, 070016 (2021); https://doi.org/10.1063/5.0057821 Published Online: 16 July 2021.
3. Suvonov O.O., Jurakulov T.T. Mathematical Modeling of Learning Processes Based on t he Theory of Control CENTRAL ASIAN JOURNAL OF MATHEMATICAL THEORY AND COMPUTER SCIENCES
4. http://cajmtcs.centralasianstudies.org/index.php/CAJMTCS Volume: 03 Issue: 04 | Apr 2022 ISSN: 2660-5309
5. Dobrynina N.F. Mathematical models of knowledge dissemination and management of student learning// Fundamental research. - 2009. - No. 7.
6. Dorrer A.G., Ivanilova T.N. Modeling an interactive adaptive training course // Modern problems of science and education. - 2007. - No. 5.
7. Ivashkin Yu.A., Nazoikin E.A. Multi-agent simulation modeling of the process of knowledge accumulation// Software products and systems. - 2011. - No. 1. – P. 47–52.
8. Kudryavtsev V.B., Vashik K., Strogalov A.S., Aliseichik P.A., Peretrukhin V.V. On automatic modeling of the learning process// Discrete Mathematics. - 1996. - V. 8, no. 4. – P. 3–10.
9. Leontiev L.P., Gokhman O.G. Problems of educational process management: Mathematical models. - Riga, 1984. - 239 p.
10. Mayer R.V. Study of the process of formation of empirical knowledge in physics. - Glazov: GSPI, 1998. - 132 p. URL: http://maierrv.glazov.net (accessed 09/27/2013).
11. Solovov A.V., Menshikov A.A. Discrete Mathematical Models in the Study of Automated Learning Processes// Educational Technology & Society. - 2001. - No. 4. – S. 205–210.