Compression curved plate loaded with compressive forces applied to unreinforced edges

[1]N.Melikulov	[1] Candidate of Technical Sciences, Associate Professor, Samarkand State Institute of Architecture and Construction (normat.meliqulov@bk.ru)
[2]S.K.Kuchkorov	[2] T.f.t.d.PhD., Associate Professor, Namangan Civil Engineering Institute (qosobirjon@gmail.com)
[3]U.Khushvaktov	[3] Senior Lecturer, Samarkand State Institute of Architecture and Construction (samgasi-ilmiy@mail.ru)
[4]O.M.Ubaydullaev	[4] Senior Lecturer, Samarkand State Institute of Architecture and Construction (hodja2002@mail.ru)

ABSTRACT

In this article, free oscillation of the plate is considered when the sides of $x=0$, a are hinged and other sides of $y=\pm\frac{b}{2}$ are free. Free sides connected with thin, open profile rods.

Keywords: compressed, compression curved, tensile, plates, elastic pinched, hinged, torsional, bending, reinforced, unreinforced, edge, rigidity

Formulation of the problem and literature review. It is known that the possibilities for enhancing the torsional stiffness of thin-walled open-profile rods can be more completely shown when limited torsion is taken into account. Therefore, when thin-walled open profile rods are used as reinforcing elements, the effect of limited (constrained) torsion is seen in an increase in the degree of pinching of the plate's edges.

The following approach of drafting the boundary conditions along the contact line of the plate with reinforcing ribs is described in the literature [2, 3, 4, 6]. The forces in the corresponding parts of the plate are thought to be equal to, but reversed in direction from, the loads transmitted by the plate to the ribs (rods). Then, into these force circumstances, are incorporated kinematic conditions of equality of displacements at the locations of contact of the reinforcing rods with the plate.

Publications [1, 5] propose a method for compiling refined boundary conditions on the line of conjugation of a plate with a rod, which makes it possible to take into account the constraint of deformation of the end sections of the ribs. In this instance, the degree of deplanation constraint is taken into consideration by some d_k parameter.

Method of solution. According to our knowledge, only when two parallel edges are hinged can a closed (silent) solution to the problem of a compressed-curved rectangular plate be found, although the other two edges can be fixed in an arbitrary way (M. Levy's solution).

Now we shall examine the problem of a compressed-curved plate with edges $x=0, a$ hinged on rigid supports, and edges $y = \pm\frac{b}{2}$ rigidly fastened with elastic thin-walled rods of an open profile (Figure 1).

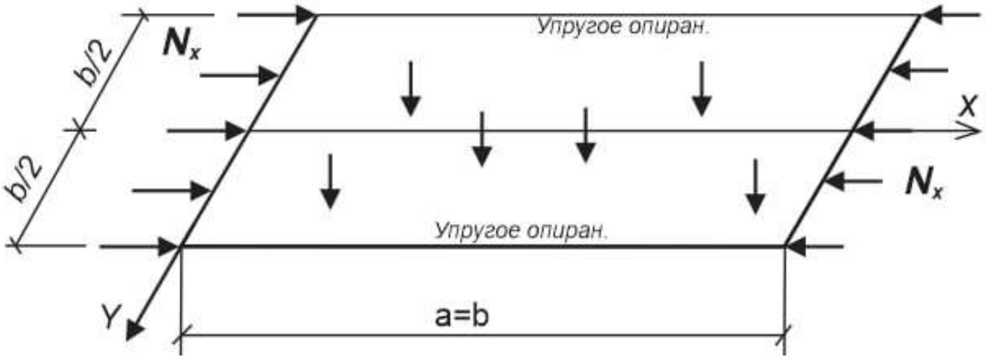

Figure 1. Compressed-curved plate

Let the load N_x be applied to the edges $x = 0, a$. In this case, the equation for the problem has the following form:

$$D\nabla^2\nabla^2 w + N_x\frac{\partial^2 w}{\partial x^2} = q(x,y) \qquad (1)$$

The solution of equation (1), which satisfies the boundary conditions of hinged support of the edges $x = 0, a$ has the below well-known form

$$w(x,y) = \sum_{n=1.2}^{\infty} f_n(y) sin\lambda_n x, \quad \lambda_n = \frac{n\pi}{a} \qquad (2)$$

Transverse load $q(x,y)$ is presented as a Fourier series in $sin\lambda_n x$ $q_n(x,y) = \sum_{n=1,2}^{\infty} q_n(y)\, sin\lambda_n x$ (3)

here the coefficients of this series are $q_n(y) = \frac{2}{a}\int_a^0 q(x,y)\, sin\lambda_n x dx$ (4)

Substituting (2) and (3) into (1) we obtain an equation for determining the function $f_n(y)$

$$f_n^{IV} - 2\lambda_n^2 f_n^{II} + \lambda_n^2(\lambda_n^2 - \frac{N_x}{D})f_n = \frac{q_n(y)}{D} \qquad (5)$$

Further, for simplicity, we apply $q = const$, then from (4) we get $q_n = \frac{4q}{n\pi}$, $(n = 1,3 \ldots.)$

The solution of equation (5) depends on the form of the roots which are characteristic to the equation. In this regard, we will consider two options below.

Option I. Let us take $N_x < \lambda_n^2 D$, here all the roots which are characteristic to the equation composed for (5) will be real, and therefore the solution can be written in the following form

$$f_n(y) = C_1 ch\alpha_n y + C_2 sh\alpha_n y + C_3 ch\beta_n y + C_4 sh\beta_n y + f_{ч.р} \qquad (6)$$

where the special solution has the form of

$$f_{ч.р} = \frac{4q}{[Dn\pi\lambda_n^2((\lambda_n^2 - \frac{N_x}{D})]} \qquad (7)$$

In (6) it is defined as $\alpha_n = \lambda_n\sqrt{1 + \sqrt{\frac{N_x}{D\lambda_n^2}}}$ $\beta_n = \lambda_n\sqrt{1 - \sqrt{\frac{N_x}{D\lambda_n^2}}}$ (8)

Option II. Now let us take $N_x > \lambda_n^2 D$, then one pair of roots which are characteristic to the equation will be real, and the second - imaginary. In this case, the solution to equation (5) has the form of

$$f_n(y) = C_1 ch\alpha_n y + C_2 sh\alpha_n y + C_3 cos\beta_n y + C_4 sin\beta_n y + f_{ч.р} \qquad (9)$$

where $\beta_n = \lambda_n\sqrt{\sqrt{\frac{N_x}{D\lambda_n^2}} - 1}$ (10)

The special solution $f_{ч.р}$ and the parameter α_n are given by formulas (7) and (8).

Option III. When $N_x = \lambda_n^2 D$ is not considered separately, as it can be arbitrarily close to it from option I or option II.

Boundary conditions for elastic pinching and elastic support of the edge is $y = \pm\frac{b}{2}$:

Conditions of elastic pinching at the edges $y = \pm\frac{b}{2}$:

$f_n^I = \frac{\mu\lambda_n^2 f_n - f_n^{II}}{t_k b \lambda_n^2}$, (11)

где $t_k = \frac{C_k}{d_k}(1 + \frac{\lambda_n^2}{k^2})$, $C_k = \frac{GJ_k}{Db}$, $k^2 = \frac{GJ_k}{EJ_\omega}$, (12)

$d_k = 1$ при $\frac{d^2\theta}{dx^2}\Big|_{x=0,a} = 0$ (13)

$d_k = 1 - \frac{2ka}{n^2\pi^2 - k^2a^2}\left\{\frac{[1-(-1)^n]\,\mathrm{sh}\,ka}{chka-1} + \frac{[1+(-1)^n]ka(1-chka)}{2(chka-1)-kashka}\right\}$ *when* $\frac{d\theta}{dx}\Big|_{x=0,a} = 0$ (13a)

Conditions of elastic pinching at the edges $y = \pm\frac{b}{2}$:

$f_n = \frac{f_n^{III} - (2-\mu)\,\lambda_n^2 f_n^I}{t_u b \lambda_n^4}$, (14)

где $t_u = \frac{C_u}{d_u(1+\lambda_n^2\frac{k_\varphi}{k_2^2})}$, $C_u = \frac{EJ_y}{Db}$, $k_2^2 = \frac{GF}{EJ_y}$, (15)

$d_u = 1$ при $\frac{d^2 w_{п.с.}}{dx^2}\Big|_{x=0,a}, = 0$, (16)

$d_u = 1 - \frac{16+8(-1)^n}{n^2\pi^2}$ when $\frac{dw_{п.с.}}{dx}\Big|_{x=0,a} = 0$. (16a)

Now we place each of the functions $f_n(y)$ based on (6) and (7) under the boundary conditions (11) and (14). To simplify the solution, we use the symmetry of the problem and set $C_2 = C_4 = 0$. As a result, we obtain the following equations for the constant C_1, C_3:

Option I.

$$\left.\begin{aligned} &C_1(A_n ch\xi + 2t_k\psi^2\xi sh\xi) + C_3\left(B_n^{(-)}ch\eta + 2t_k\psi^2\eta sh\eta\right) = \mu\psi^2 f_{ч.р} \\ &C_1(t_u\psi^4 ch\xi - 2C_n\xi sh\xi) + C_3\left(t_u\psi^4 ch\eta - 2D_n^{(-)}\eta sh\eta\right) = -t_u\psi^4 f_{ч.р} \end{aligned}\right] \quad (17)$$

Option II.

$$\left.\begin{aligned} &C_1(A_n ch\xi + 2t_k\psi^2\xi sh\xi) - C_3(B_n cos\eta + 2t_k\psi^2\eta sin\eta) = \mu\psi^2 f_{ч.р} \\ &C_1(t_u\psi^4 ch\xi - 2C_n\xi sh\xi) + C_3(t_u\psi^4 cos\eta - 2D_n\eta sh\eta) = -t_u\psi^4 f_{ч.р} \end{aligned}\right] \quad (18)$$

In (17), (18) the following indication is presented

$A_n = 4\xi^2 - \mu\psi^2$, $B_n = 4\eta^2 + \mu\psi^2$, $\xi = \frac{\alpha_n b}{2}$.

$B_n^{(-)} = 4\eta^2 - \mu\psi^2$, $C_n = 4\xi^2 - (2-\mu)\psi^2$, $\eta = \frac{\beta_n b}{2}$. (19)

$D_n = 4\eta^2 + (2-\mu)\psi^2$, $D_n^{(-)} = 4\eta^2 - (2-\mu)\psi^2$, $\psi = \lambda_n b$

Having found constant C_1, C_3 from (17) and (18), we can present the expression for deflection (2) in the final form of

option I ($N_x < \lambda_n^2 D$)

$$w(x,y) = \frac{4qb^4}{\pi^5 D}\sum_{n=1.3..}^{\infty}\frac{sin\lambda_n x}{n^5\left(1 - \frac{4N_x a^2}{N_э n^2 b^2}\right)} \times$$

$$\times\{1 - \frac{\psi^2[(B_1 t_u\psi^2 + B_2\mu)ch\beta_n y - (B_3 t_u\psi^2 + B_4\mu)ch\alpha_n y]}{B_1B_4 - B_2B_3}\} \quad (20)$$

где

$B_1 = (4\xi^2 - \mu\psi^2)ch\xi + 2t_k\psi^2\xi sh\xi$,

$B_2 = t_u\psi^4 ch\xi - 2[4\xi^2 - (2-\mu)\psi^2]\xi sh\xi$,

$B_3 = (4\eta^2 - \mu\psi^2)ch\eta + 2t_k\psi^2\eta sh\eta$, (21)

$B_4 = t_u\psi^4 ch\eta - 2[4\eta^2 - (2-\mu)\psi^2]\eta sh\eta.$

Option II ($N_x > \lambda_n^2 D$): It is summarized to formula (20) by replacing the last expression in curly brackets with the following expression

$$1 - \frac{\psi^2[(B_1 t_u\psi^2 + B_2\mu)cos\beta_n y + (B_5 t_u\psi^2 + B_6\mu)ch\alpha_n y]}{B_1B_6 + B_2B_5} \qquad (22)$$

where $B_5 = (4\eta^2 + \mu\psi^2)cos\eta + 2t_k\psi^2\eta sin\eta,$

$$B_6 = t_u\psi^4 cos\eta - 2[4\eta^2 + (2-\mu)\psi^2]\eta sh\eta] \qquad (23)$$

Let us examine special cases when the plate is pivotally attached to elastic reinforcing rods, that is $t_k = 0, 0 \le t_u \le \infty$

Formulas for deflections (20) and (22) for this case will take the form of Option I.

$$w(x,y) = \frac{4qb^4}{\pi^5 D}\sum_{n=1.3..}^{\infty}\frac{sin\lambda_n x}{n^5\left(1 - \frac{4N_x a^2}{N_э n^2 b^2}\right)}\times$$

$$\times\left(1 - \frac{\psi^2[(B_7 t_u\psi^2 + B_2\mu)ch\beta_n y - (B_8 t_u\psi^2 + B_4\mu)ch\alpha_n y]}{B_4B_7 - B_2B_8}\right). \qquad (24)$$

Option II.

$$w(x,y) = \frac{4qb^4}{\pi^5 D}\sum_{n=1.3..}^{\infty}\frac{sin\lambda_n x}{n^5\left(1 - \frac{4N_x a^2}{N_э n^2 b^2}\right)}\times$$

$$\times\left(1 - \frac{\psi^2[(B_7 t_u\psi^2 + B_2\mu)cos\beta_n y + (B_9 t_u\psi^2 - B_6\mu)ch\alpha_n y]}{B_6B_7 + B_2B_9}\right) \qquad (25)$$

In (24) and (25) the following indication is presented

$B_7 = (4\xi^2 - \mu\psi^2)ch\xi,\ B_8 = (4\eta^2 - \mu\psi^2)ch\eta\ , B_9 = (4\eta^2 + \mu\psi^2)cos\eta$ (26)

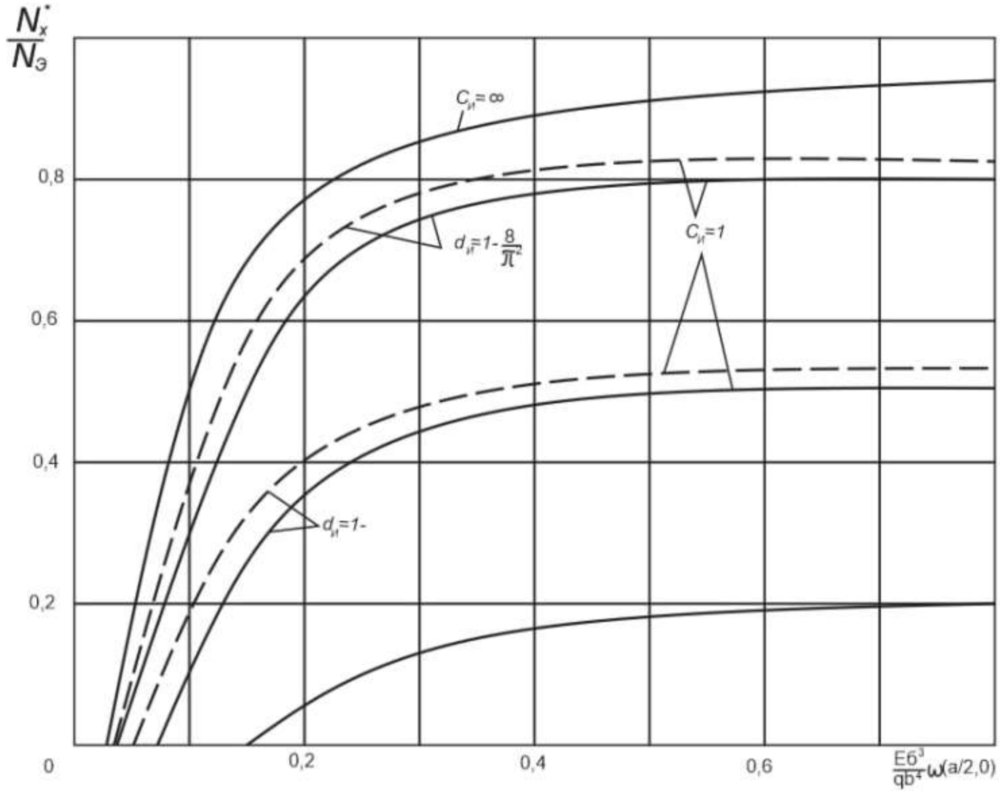

Figure 2. Graph of the dependence of the deflection in the center of the plate on the data N_x.

Figure 2 illustrates, according to formulas (24) and (25), a graph of the dependence of the deflection in the center of the plate on the data N_x. In construction used formula $a = b$, $n = 1$, $P = 0$, $\mu = 0{,}3$. Solid lines refer to cases when shear deformation is taken into account during bending of reinforcing bars $k_2 a = 10$). Dashed lines are constructed without taking into account shear ($k_2 a = \infty$). From the graph it can be seen that the differences in the boundary conditions at the ends of the rods (hinged support, i.e. $d_u = 1$ or rigid pinching, i.e. $d_u = 1 - \frac{8}{\pi^2}$) significantly affects the deflection level. Thus, Figure 2 reveals that in order to achieve the same value of relative deflection in the center of the plate (for example, 0.2) with hinged fastening and pinching of the ends of the reinforcing rods, it is necessary in the latter case to increase the compressive force by about 2 times.

Conclusion

For square (or nearly square) plates, there is little effect from accounting for shear deformation during the bending of the reinforcing bars on the intensity of elastic pinching of the edges. However, for extended plates, accounting for the aforementioned issue can modify the qualitative perception of the plate curvature as well as drastically reduce the critical load parameters (change the number of half-waves).

Reference books

1. Beilin E.A., Melikulov N.M. On the stability of rectangular plates reinforced with thin-walled rods. -In book: Structural mechanics and calculation of structures: Scientific and technical journal. M. Publishing house of literature on construction. 1980, No. 5. pp. 38-42.
2. Bleykh F. Stability of metal structures. M. Fizmatgiz. 1959.
3. Broude B.M. Stability of plates in elements of steel structures M.Mashstroyizdat. 1949.
4. Volmir A.S. Stability of elastic systems. M.Phys.Mat.Giz.1967.
5. Melikulov N.M. Study of the stability and rigidity of plates reinforced with thin-walled rods under various loading cases, - In the book: Structural mechanics of structures. Inter-university theme collection. LISI, 1980. pp. 76-85.
6. Tymoshenko S.P. Stability of elastic systems. M. Gostekhizdat, 1955.
7. Muszkowska H. Plyty prostokatne o dwoch krawedziach przeciwleglych swobodnie podparnech i pozostalech sprezyscie zamocowanech. Prace Naykowe Insnytutu Budownictwa Politechniki Wroclawskcy.1973, Nr.11.
8. Ferachian R.H. Back ling of biaxial le compressed long rectangular plates elastically restrained along the long edges and simply supported along the short edges. Proc. Inst. Engrs. Part 2. Montreal, 1975.
9. Melikulov N,. Khodjabekov, M. U.Ismatova D. M. Otaqylov A. FREE VIBRATIONS OF THE PLATE WITH THE ACCOUNT OF INFLUENCE OF LONGITUDINAL FORCES PERCEIVED BY THE REINFORCING RODS. European Journal of Research. Volume 5, issue 8, 2020. pp. 20- 25.