Numerical Solution of a Two-Dimensional Dynamic Related Problem of Thermal Support

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ABSTRACT

Related thermodynamic elastic boundary problems based on Ilyushin's theory of deformation have been formulated. Two types of explicit and implicit two-dimensional schemes have been constructed using the finite difference method. In the case of an explicit scheme, the numerical solution of the problem is based on a recurrence relation. In the case of an implicit scheme, the method of solving the problem is reduced to the use of the "run" method. Comparison of the numerical results obtained by the two methods shows that they are quite close.

Keywords: Deformation, displacement, temperature, heat resistance, thermoplastic, bound, unbound.

Introduction. The study of thermoplastic states of structures and their elements is an urgent problem of the mechanics of a deformable solid. When staging thermoelastic [1] problems, related and unbound boundary sets are distinguished. In the general case of the related problem, the equations of motion of a solid body are considered in combination with the equation of thermal conductivity. It should be noted that temperature and its the derivative is included in the equation of motion, and the deformation is included in the equation of thermal conductivity.

In general, the thermomechanical boundary problem of the mechanics of a deformable solid consists of an equation of motion that determines the relations of thermoelasticity [2], the Cauchy relation, the equations of heat inflow with the corresponding initial and boundary conditions. Note that in this case, the equations of motion written in the displacements and the equation of heat inflow are related, i.e. temperature as an unknown function of the superintendent. Note that in this case, the equations of motion recorded in the displacements and the equation of heat inflow are related, i.e. temperature as an unknown function of the superformation it is included in the equation of motion, and the equation of heat inflow depends on the displacement [3-5].

If the external factors that cause the motion of the body change very slowly in time, then in the equation of motion inertial terms can be neglected, treating the problem as quasi-static [5]. At the same time, the initial conditions for displacement disappear, but the quasi-static problem remains related. If the quantities causing the deformation and temperature change slowly enough from zero to their final values and remain in this state, then we get a static problem. Displacement and temperature become timeless and are functions of the coordinates of the position of points and in the equations the terms containing derivatives of time disappear. In this case, we have an unrelated thermoplasticity problem [1].

Problem statement. The related thermodynamic marginal problem of thermoelasticity consists of equations of motion [2]

$$\sigma_{ij,j} + X_i = \rho\ddot{u}_i, \qquad (1)$$

of the determining relation of the deformation theory of A.A. Ilyushin [3]

$$\sigma_{ij} = K(\theta - 3\alpha\vartheta)\delta_{ij} + \frac{\sigma_u}{\varepsilon_u}e_{ij}, \quad K = \lambda + \frac{2}{3}\mu, \quad \vartheta = T - T_0 \qquad (2)$$

heat inflow equations for isotropic materials [5]

$$\lambda_0 T_{,ii} - C_\varepsilon\dot{T} - \alpha\gamma T_0 \cdot \dot{\varepsilon}_{ij} = 0, \qquad (3)$$

Cauchy ratios $$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + v_{j,i}\right), \qquad (4)$$

with beginner $$u_i\big|_{t=0} = \varphi_i,\ \dot{u}_i\big|_{t=0} = \psi_i,\ T\big|_{t=0} = T_0, \qquad (5)$$

and Edge Conditions $$u_i\big|_{\Sigma_1} = u_i^0, \quad T\big|_{\Sigma_1} = \overline{T}_0, \quad \sigma_{ij}n_j\big|_{\Sigma_2} = S_i^0, \qquad (6)$$

The equations of thermoelasticity and the equation of thermal conductivity in the two-dimensional case take the form [7-9]:

$$\left(\lambda + 2\mu\right)\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 u}{\partial y^2} + \left(\lambda + \mu\right)\frac{\partial^2 v}{\partial x\partial y} - \alpha(3\lambda + 2\mu)\left(\frac{\partial T}{\partial x}\right) = \rho\frac{\partial^2 u}{\partial t^2}, \qquad (7)$$

$$\mu\frac{\partial^2 v}{\partial x^2} + \left(\lambda + 2\mu\right)\frac{\partial^2 v}{\partial y^2} + \left(\lambda + \mu\right)\frac{\partial^2 u}{\partial x\partial y} - \alpha(3\lambda + 2\mu)\left(\frac{\partial T}{\partial y}\right) = \rho\frac{\partial^2 v}{\partial t^2}, \qquad (8)$$

$$\lambda_{11}\frac{\partial^2 T}{\partial x^2} + \lambda_{22}\frac{\partial^2 T}{\partial y^2} - C_\varepsilon\frac{\partial T}{\partial t} - T(\beta_{11}\frac{\partial^2 u}{\partial x\partial t} + \beta_{22}\frac{\partial^2 v}{\partial y\partial t}) = 0. \qquad (9)$$

with appropriate initials:

$$u\left(x,y,t\right)\Big|_{t=0} = \phi_1,,, \quad \frac{\partial u}{\partial t}\Big|_{t=0} = \psi_1 \quad v\left(x,y,t\right)\Big|_{t=0} = \phi_2$$

$$\frac{\partial v}{\partial t}\Big|_{t=0} = \psi_2, \quad T\left(x,y,t\right)\Big|_{t=0} = T_0; \qquad (10)$$

and boundary conditions:

$$u\left(x,y,t\right)\Big|_{x=0} = u_0,,, \quad u\left(x,y,t\right)\Big|_{x=\ell_1} = \bar{u}_0 \quad u\left(x,y,t\right)\Big|_{y=0} = u_0' \ u\left(x,y,t\right)\Big|_{y=\ell_2} = \bar{u}_0',,$$

$$v\left(x,y,t\right)\Big|_{x=0} = v_0 \quad v\left(x,y,t\right)\Big|_{x=\ell_1} = \bar{v}_0,$$

$$v\left(x,y,t\right)\Big|_{y=0} = v_0', \quad v\left(x,y,t\right)\Big|_{y=\ell_2} = \bar{v}_0',, \quad T\left(x,y,t\right)\Big|_{x=0} = T_1\left(t\right) \qquad (11)$$

$$T(x,y,t)\big|_{x=\ell_1}=T_2(t),, \quad T(x,y,t)\big|_{y=0}=T_1'(t) \qquad T(x,y,t)\big|_{y=\ell_2}=T_2'(t);$$

gde $C_\varepsilon -$ coefficient of heat capacity at constant temperature, coefficient of thermal expansion, $\alpha -$ $\lambda,\mu -$ elastic constant, λ_{11}, $\lambda_{22} -$ coefficient of heat flux, β_{11}, $\beta_{22} -$ tensor of thermal expansion, dimensions of $\ell_1, \ell_2 -$ the plate, $\rho -$ density of the material, $\phi_1,\phi_2,\psi_1,\psi_2,u_0,u_0',\bar{u}_0,\bar{u}_0',v_0,v_0',\bar{v}_0,\vec{v}_0',T_0,T_1,T_2,T_1',T_2' -$ specified values.

Numerical implementation. Having built into $t \ge 0, \quad 0 \le x \le \ell_1, \quad 0 \le y \le \ell_2$ three families of parallel lines, we replace $x_i = ih_1 \quad (i=\overline{0,n}),\ y_j = jh_2 \quad (j=\overline{0,m}),\ t=k\tau \quad (k=0,1,2,\ldots)$ the derivatives in equations (7) - (9) with difference relations and solving the obtained difference equations relative to the correspondingly we get [6, 10-12] $u_{i,j}^{k+1}$, $v_{i,j}^{k+1}$, $T_{i,j}^{k+1}$

$$u_{i,j}^{k+1}=\frac{\tau^2}{\rho}\left(\mu\frac{u_{i,j+1}^k-2u_{i,j}^k+u_{i,j-1}^k}{h_2^2}+(\lambda+\mu)\frac{v_{i+1,j+1}^k-v_{i-1,j+1}^k-v_{i+1,j-1}^k+v_{i-1,j-1}^k}{4h_1h_2}+\right.$$
$$\left.+(\lambda+2\mu)\frac{u_{i+1,j}^k-2u_{i,j}^k+u_{i-1,j}^k}{h_1^2}-\alpha(3\lambda+2\mu)\frac{T_{i+1,j}^k-T_{i-1,j}^k}{2h_1}\right)+2u_{i,j}^k-u_{i,j}^{k-1}, \qquad (12)$$

$$v_{i,j}^{k+1}=\frac{\tau^2}{\rho}\left(\mu\frac{v_{i+1,j}^k-2v_{i,j}^k+v_{i-1,j}^k}{h_1^2}+(\lambda+\mu)\frac{u_{i+1,j+1}^k-u_{i-1,j+1}^k-u_{i+1,j-1}^k+u_{i-1,j-1}^k}{4h_1h_2}+\right.$$
$$\left.+(\lambda+2\mu)\frac{v_{i,j+1}^k-2v_{i,j}^k+u_{i,j-1}^k}{h_2^2}-\alpha(3\lambda+2\mu)\frac{T_{i,j+1}^k-T_{i,j-1}^k}{2h_2}\right)+2v_{i,j}^k-v_{i,j}^{k-1}, \qquad (13)$$

$$T_{i,j}^{k+1}=\frac{\tau}{C_\varepsilon}\left(\lambda_{11}\frac{T_{i+1,j}^k-Tu_{i,j}^k+T_{i-1,j}^k}{h_1^2}+\lambda_{22}\frac{T_{i,j+1}^k-2T_{i,j}^k+T_{i,j-1}^k}{h_2^2}-\right.$$
$$\left.-T_{i,j}^k\left(\beta_{11}\frac{u_{i+1,j}^{k+1}-u_{i-1,j}^{k+1}-u_{i+1,j}^{k-1}+u_{i-1,j}^{k-1}}{4h_1\tau}+\beta_{22}\frac{v_{i,j+1}^{k+1}-v_{i,j-1}^{k+1}-v_{i,j+1}^{k-1}+v_{i,j-1}^{k-1}}{4h_2\tau}\right)+T_{i,j}^k. \qquad (14)$$

As can be seen, equations (12) - (14) allow you to find the values of $u(x,y,t), v(x,y,t)$ functions $T(x,y,t)$ on the layer t^{k+1} if the values on the two previous layers are known. By replacing the index k by k+1 in the first term of the difference equation (7-9), it is possible to obtain an implicit difference scheme solved by the run-through method [13]

$$a_{i,j}u_{i+1,j}^{k+1}+b_{i,j}u_{i,j}^{k+1}+c_{i,j}u_{i-1,j}^{k+1}=f_{i,j} \qquad (15)$$

$$a_{i,j}=\frac{\lambda+2\mu}{h_1^2}, \qquad b_{i,j}=-2\frac{(\lambda+2\mu)}{h_1^2}-\frac{\rho}{\tau^2}, \qquad c_{i,j}=\frac{\lambda+2\mu}{h_1^2},$$

$$f_{i,j}=\alpha\gamma\frac{T_{i+1,j}^{k-1}-T_{i-1,j}^{k-1}}{2h_1}-\mu\frac{u_{i,j+1}^k-2u_{i,j}^k+u_{i,j-1}^k}{h_2^2}-$$
$$-(\lambda+\mu)\frac{v_{i+1,j+1}^k-v_{i+1,j-1}^k-v_{i-1,j+1}^k+v_{i-1,j-1}^k}{4h_1h_2}+\rho\frac{u_{i,j}^{k-1}-2u_{i,j}^k}{\tau^2}.$$

$$A_{i,j}v_{i+1,j}^{k+1}+B_{i,j}v_{i,j}^{k+1}+C_{i,j}v_{i-1,j}^{k+1}=F_{i,j}, \qquad (16)$$

Where is

$$A_{i,j}=\frac{\mu}{h_1^2},\quad B_{i,j}=-2\frac{\mu}{h_1^2}-\frac{\rho}{\tau^2},\quad C_{i,j}=\frac{\mu}{h_1^2},$$

$$F_{i,j}=\alpha\gamma\frac{T_{i+1,j}^{k-1}-T_{i-1,j}^{k-1}}{2h_2}-(\lambda+2\mu)\frac{v_{i,j+1}^{k}-2v_{i,j}^{k}+v_{i,j-1}^{k}}{h_2^2}-$$

$$-(\lambda+\mu)\frac{u_{i+1,j+1}^{k}-u_{i+1,j-1}^{k}-u_{i-1,j+1}^{k}+u_{i-1,j-1}^{k}}{4h_1h_2}+\rho\frac{v_{i,j}^{k-1}-2v_{i,j}^{k}}{\tau^2}.$$

Similarly, equation (9) can be brought to the form

$$a_{i,j}T_{i+1,j}^{k+1}+b_{i,j}T_{i,j}^{k+1}+c_{i,j}T_{i-1,j}^{k+1}=f_{i,j}, \tag{17}$$

Where is

$$a_{i,j}=\frac{\lambda_{11}}{h_1^2},\quad b_{i,j}=-2\frac{\lambda_{11}}{h_1^2}-\frac{C_\varepsilon}{2\tau},\quad c_{i,j}=\frac{\lambda_{11}}{h_1^2},$$

$$f_{i,j}=\beta_{11}T_0\frac{u_{i+1,j}^{k}-u_{i-1,j}^{k}-u_{i+1,j}^{k-2}+u_{i-1,j}^{k-2}}{4h_1\tau}-\lambda_{22}\frac{T_{i,j+1}^{k}-2T_{i,j}^{k}+T_{i,j-1}^{k}}{h_2^2}+$$

$$+\beta_{22}T_0\frac{v_{i,j+1}^{k}-v_{i,j-1}^{k}-v_{i,j+1}^{k-2}+v_{i,j-1}^{k-2}}{4h_2\tau}-\frac{C_\varepsilon}{2\tau}T_{i,j}^{k-1}.$$

The $u(x,y,t)$ values of the functions are also on the two initial layers $v(x,y,t)$ and $k=0$ we will find from the initial conditions, and for the value $k=1$ of the function we will find replacing the mixed derivatives with other difference relations $T(x,y,t)$ [14-16].

$$u_{i,j}^1=\left(\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{u_{i+1,j}^0-2u_{i,j}^0+u_{i-1,j}^0}{h_1^2}+\mu\frac{u_{i,j+1}^0-2u_{i,j}^0+u_{i,j-1}^0}{h_2^2}+\right.\right.$$

$$\left.+(\lambda+\mu)\frac{v_{i+1,j+1}^0-v_{i-1,j+1}^0-v_{i+1,j-1}^0+v_{i-1,j-1}^0}{4h_1h_2}-\alpha(3\lambda+2\mu)\frac{T_{i+1,j}^0-T_{i-1,j}^0}{2h_1}\right)+ \tag{18}$$

$$\left.+2u_{i,j}^0+2\tau\psi_1\right)/2,$$

$$v_{i,j}^1=\left(\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{v_{i,j+1}^0-2v_{i,j}^0+u_{i,j-1}^0}{h_2^2}+\mu\frac{v_{i+1,j}^0-2v_{i,j}^0+v_{i-1,j}^0}{h_1^2}+\right.\right.$$

$$\left.+(\lambda+\mu)\frac{u_{i+1,j+1}^0-u_{i-1,j+1}^0-u_{i+1,j-1}^0+u_{i-1,j-1}^0}{4h_1h_2}-\alpha(3\lambda+2\mu)\frac{T_{i,j+1}^0-T_{i,j-1}^0}{2h_2}\right)+ \tag{19}$$

$$\left.+2v_{i,j}^0+2\tau\psi_2\right)/2,$$

$$T_{i,j}^1=\frac{\tau}{C_\varepsilon}\left(\lambda_{11}\frac{T_{i+1,j}^0-2T_{i,j}^0+T_{i-1,j}^0}{h_1^2}+\lambda_{22}\frac{T_{i,j+1}^0-2T_{i,j}^0+T_{i,j-1}^0}{h_2^2}-\right.$$

$$\left.-T_{i,j}^0\left(\beta_{11}\frac{u_{i+1,j}^1-u_{i-1,j}^1-u_{i+1,j}^0+u_{i-1,j}^0}{2h_1\tau}+\beta_{22}\frac{v_{i,j+1}^1-v_{i,j-1}^1-v_{i,j+1}^0+v_{i,j-1}^0}{2h_2\tau}\right)\right)+T_{i,j}^0. \tag{20}$$

The values of displacements and temperatures starting from the second layer can be found by $u_{i,j}^{k+1}$, $v_{i,j}^{k+1}$ $T_{i,j}^{k+1}$ the distillation method (15) - (17), and the values of these functions on the first layer are found according to the method considered in [10].

Test task. The related problem of thermoelasticity (12-17) has been solved by explicit (by the method of grids) and implicit (by the method of running) method. Copper is considered as a material under the following initial and boundary conditions:

$$u(x,y,t)\big|_{t=0}=0,\qquad \left.\frac{\partial u(x,y,t)}{\partial t}\right|_{t=0}=0,\qquad v(x,y,t)\big|_{t=0}=0,\qquad \left.\frac{\partial v(x,y,t)}{\partial t}\right|_{t=0}=0,$$

$$T(x,y,t)\big|_{x=0/x=1}=T_1\sin(\pi y/\ell_2),\quad T(x,y,t)\big|_{y=0/y=1}=0,\quad T(x,y,t)\big|_{t=0}=T_0;$$

the following elastic constants are used: $\lambda=1,\ \lambda_{11}=0.4,\ \lambda_{22}=0.4,\ \alpha=0.093,\ \beta_{11}=0.15,\ \beta_{22}=0.15,$ $\mu=0.5,\ \rho=8.9,\ C_{\varepsilon}=0.38,\ T_0=20,\ T_1=60,\ h_1=0.1,\ h_2=0.1,\ \tau=0.01,\ n=10,\ \ell_1=1,\ \ell_2=1;$

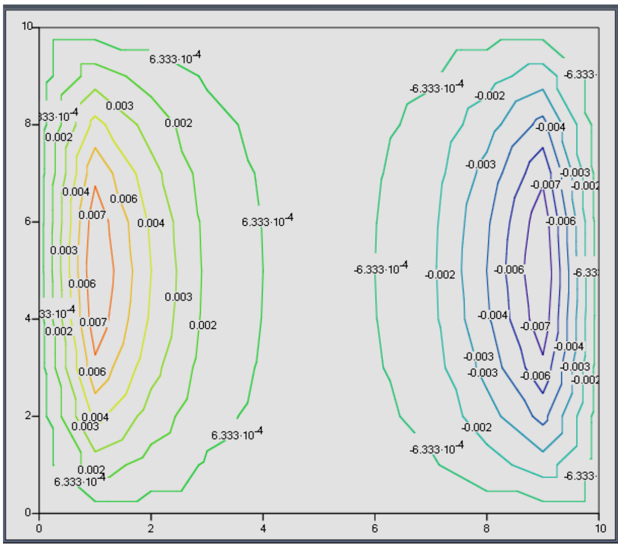

Рис. 1. Перемещения U(x, y, tn) (метод прогонки)

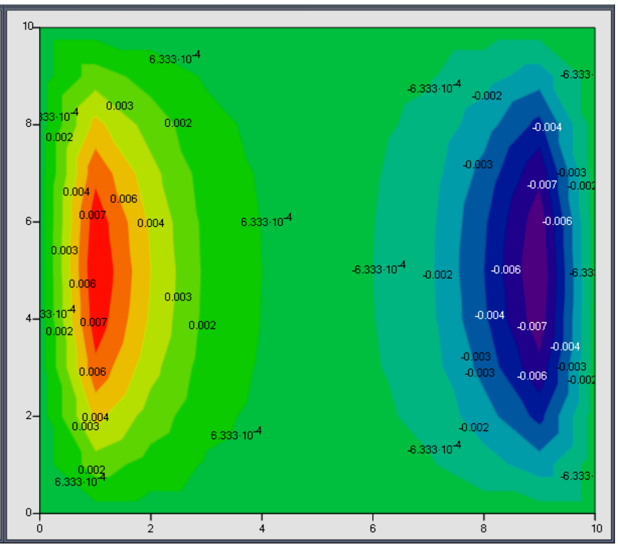

Рис. 2. Перемещения U(x, y, tn) (метод сеток)

Перемещение U(x, y, tn) (метод прогонки)Таблица 1.

x \ and	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
0	0	0	0	0	0	0	0	0	0	0	0
0,1	0	0,0024	0,0014	0,0006	0,0002	0	-0,0002	-0,0006	-0,0014	-0,0024	0
0,2	0	0,0046	0,0026	0,0012	0,0005	0	-0,0005	-0,0012	-0,0026	-0,0046	0
0,3	0	0,0064	0,0036	0,0016	0,0006	0	-0,0006	-0,0016	-0,0036	-0,0064	0
0,4	0	0,0075	0,0042	0,0019	0,0007	0	-0,0007	-0,0019	-0,0042	-0,0075	0

0,5	0	0,0079	0,0044	0,002	0,0008	0	-0,0008	-0,002	-0,0044	-0,0079	0
0,6	0	0,0075	0,0042	0,0019	0,0007	0	-0,0007	-0,0019	-0,0042	-0,0075	0
0,7	0	0,0064	0,0036	0,0016	0,0006	0	-0,0006	-0,0016	-0,0036	-0,0064	0
0,8	0	0,0046	0,0026	0,0012	0,0005	0	-0,0005	-0,0012	-0,0026	-0,0046	0
0,9	0	0,0024	0,0014	0,0006	0,0002	0	-0,0002	-0,0006	-0,0014	-0,0024	0
1	0	0	0	0	0	0	0	0	0	0	0

Перемещение U(x, y, t_n) (метод сеток)Таблица 2.

x / and	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
0	0	0	0	0	0	0	0	0	0	0	0
0,1	0	0,0025	0,0014	0,0005	0,0002	0	-0,0002	-0,0005	-0,0014	-0,0025	0
0,2	0	0,0048	0,0026	0,001	0,0004	0	-0,0004	-0,001	-0,0026	-0,0048	0
0,3	0	0,0067	0,0036	0,0013	0,0005	0	-0,0005	-0,0013	-0,0036	-0,0067	0
0,4	0	0,0078	0,0042	0,0015	0,0006	0	-0,0006	-0,0015	-0,0042	-0,0078	0
0,5	0	0,0082	0,0044	0,0016	0,0006	0	-0,0006	-0,0016	-0,0044	-0,0082	0
0,6	0	0,0078	0,0042	0,0015	0,0006	0	-0,0006	-0,0015	-0,0042	-0,0078	0
0,7	0	0,0067	0,0036	0,0013	0,0005	0	-0,0005	-0,0013	-0,0036	-0,0067	0
0,8	0	0,0048	0,0026	0,001	0,0004	0	-0,0004	-0,001	-0,0026	-0,0048	0
0,9	0	0,0025	0,0014	0,0005	0,0002	0	-0,0002	-0,0005	-0,0014	-0,0025	0
1	0	0	0	0	0	0	0	0	0	0	0

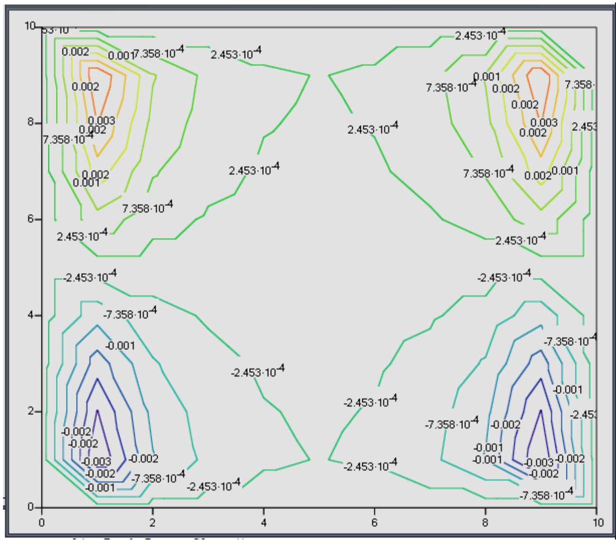

Рис. 3. Перемещения V(x, y, tn) (метод прогонки)

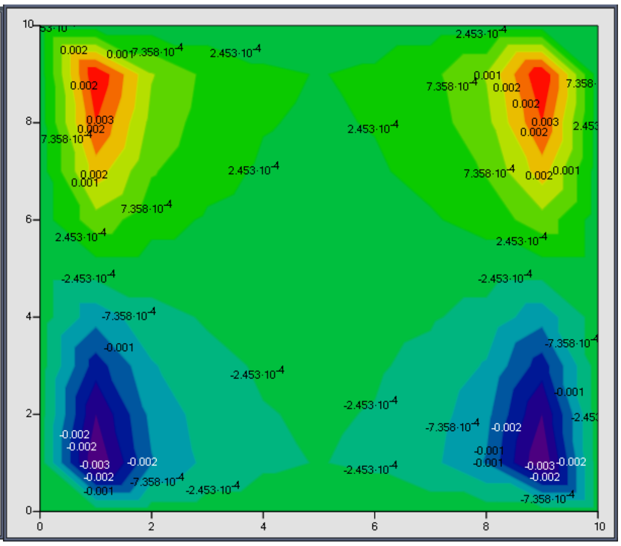

Рис. 4. Перемещения V(x, y, tn) (метод сеток)

Перемещение V(x, y, tn) (метод прогонки) Таблица 3.

x and	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
0	0	0	0	0	0	0	0	0	0	0	0
0,1	0	-0,0032	-0,0013	-0,0006	-0,0003	-0,0002	-0,0003	-0,0006	-0,0013	-0,0032	0
0,2	0	-0,0027	-0,0011	-0,0005	-0,0003	-0,0002	-0,0003	-0,0005	-0,0011	-0,0027	0
0,3	0	-0,002	-0,0008	-0,0004	-0,0002	-0,0001	-0,0002	-0,0004	-0,0008	-0,002	0
0,4	0	-0,001	-0,0004	-0,0002	-0,0001	-0,0001	-0,0001	-0,0002	-0,0004	-0,001	0
0,5	0	0	0	0	0	0	0	0	0	0	0
0,6	0	0,001	0,0004	0,0002	0,0001	0,0001	0,0001	0,0002	0,0004	0,001	0
0,7	0	0,002	0,0008	0,0004	0,0002	0,0001	0,0002	0,0004	0,0008	0,002	0
0,8	0	0,0027	0,0011	0,0005	0,0003	0,0002	0,0003	0,0005	0,0011	0,0027	0
0,9	0	0,0032	0,0013	0,0006	0,0003	0,0002	0,0003	0,0006	0,0013	0,0032	0
1	0	0	0	0	0	0	0	0	0	0	0

Перемещение V(x, y, tn) (метод сеток)
Таблица 4.

x and	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
0	0	0	0	0	0	0	0	0	0	0	0

0,1	0	-0,0032	-0,0015	-0,0006	-0,0003	-0,0002	-0,0003	-0,0006	-0,0015	-0,0032	0
0,2	0	-0,0027	-0,0012	-0,0005	-0,0002	-0,0002	-0,0002	-0,0005	-0,0012	-0,0027	0
0,3	0	-0,002	-0,0009	-0,0004	-0,0002	-0,0001	-0,0002	-0,0004	-0,0009	-0,002	0
0,4	0	-0,001	-0,0005	-0,0002	-0,0001	-0,0001	-0,0001	-0,0002	-0,0005	-0,001	0
0,5	0	0	0	0	0	0	0	0	0	0	0
0,6	0	0,001	0,0005	0,0002	0,0001	0,0001	0,0001	0,0002	0,0005	0,001	0
0,7	0	0,002	0,0009	0,0004	0,0002	0,0001	0,0002	0,0004	0,0009	0,002	0
0,8	0	0,0027	0,0012	0,0005	0,0002	0,0002	0,0002	0,0005	0,0012	0,0027	0
0,9	0	0,0032	0,0015	0,0006	0,0003	0,0002	0,0003	0,0006	0,0015	0,0032	0
1	0	0	0	0	0	0	0	0	0	0	0

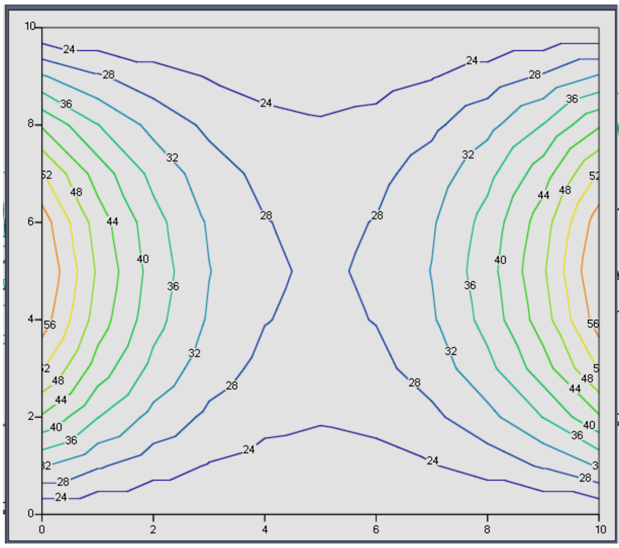

Рис. 5. Температура T(x, y, tn) (метод прогонки)

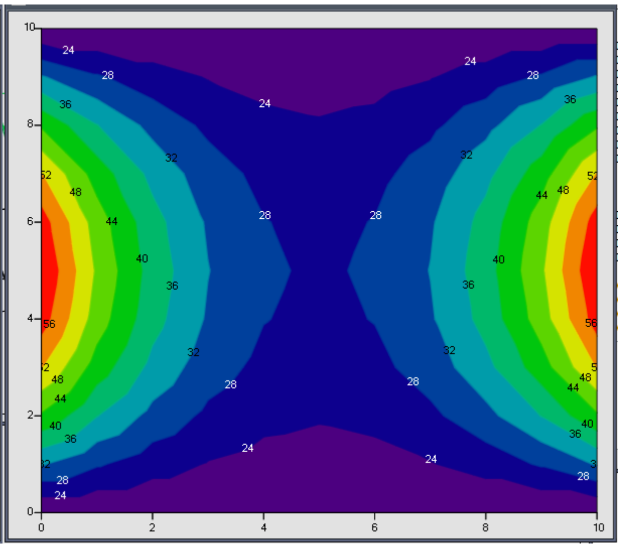

Рис. 6. Температура T(x, y, tn) (метод сеток)

Temperature T(x, y, tn) (banishing method)Table 5.

x \ and	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
0	20	20	20	20	20	20	20	20	20	20	20
0,1	32,36	28,5	25,68	23,76	22,65	22,29	22,65	23,76	25,68	28,5	32,36
0,2	43,51	36,13	30,78	27,13	25,04	24,36	25,04	27,13	30,78	36,13	43,51

0,3	52,36	42,2	34,82	29,81	26,93	25,99	26,93	29,81	34,82	42,2	52,36
0,4	58,04	46,1	37,43	31,54	28,15	27,05	28,15	31,54	37,43	46,1	58,04
0,5	60	47,44	38,32	32,13	28,57	27,41	28,57	32,13	38,32	47,44	60
0,6	58,04	46,1	37,43	31,54	28,15	27,05	28,15	31,54	37,43	46,1	58,04
0,7	52,36	42,2	34,82	29,81	26,93	25,99	26,93	29,81	34,82	42,2	52,36
0,8	43,51	36,13	30,78	27,13	25,04	24,36	25,04	27,13	30,78	36,13	43,51
0,9	32,36	28,5	25,68	23,76	22,65	22,29	22,65	23,76	25,68	28,5	32,36
1	20	20	20	20	20	20	20	20	20	20	20

Temperature $T(x, y, t_n)$ (grid method)Table 6.

X \ T	0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1
0	20	20	20	20	20	20	20	20	20	20	20
0,1	32,36	28,31	26,32	23,14	23,64	21,45	23,64	23,14	26,32	28,31	32,36
0,2	43,51	35,47	32,44	25,6	27,15	22,58	27,15	25,6	32,44	35,47	43,51
0,3	52,36	41,56	36,74	28	29,66	23,67	29,66	28	36,74	41,56	52,36
0,4	58,04	45,18	39,89	29,24	31,44	24,26	31,44	29,24	39,89	45,18	58,04
0,5	60	46,57	40,8	29,79	32	24,5	32	29,79	40,8	46,57	60
0,6	58,04	45,18	39,89	29,24	31,44	24,26	31,44	29,24	39,89	45,18	58,04
0,7	52,36	41,56	36,74	28	29,66	23,67	29,66	28	36,74	41,56	52,36
0,8	43,51	35,47	32,44	25,6	27,15	22,58	27,15	25,6	32,44	35,47	43,51
0,9	32,36	28,31	26,32	23,14	23,64	21,45	23,64	23,14	26,32	28,31	32,36
1	20	20	20	20	20	20	20	20	20	20	20

Fig.1 - Fig.4 shows the distribution of movements and in Fig.5 and Fig.6 the temperature distribution built on the results of explicit and implicit schemes. Tables 1-6 show the numerical results of the thermoelastic problem according to explicit (grid method) and implicit schemes (run-through method). Comparison of the results regarding movements , and temperature shows, and from the figures it can be seen that the numerical results obtained from the explicit (grid method) and implicit (run-through method) schemes are almost the same and close enough to ensure the reliability of the results obtained. $u(x, y, t)$ $v(x, y, t)$, $\mathrm{T}(x, y, t)$ $u(x, y, t)$ $v(x, y, t)$ $T(x, y, t)$

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