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Hermitian Spline in E4 Space

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ABSTRACT

The actual problem of computer-aided design in the creation of effective mathematical models of complex objects of modern construction and technology, which improves the quality and reduces the development time of projects, and also relevant is the development of scientific foundations for constructing computer graphics tools, methods for geometric modeling of designed objects. This article is devoted to this problem, the results of the research of which contain the solution of the issues of geometric modeling of designed objects, as well as mathematical software for use with computer graphics.

Keywords: Hypernets, finite difference method, interpolation, boundary conditions, membership, bypass points, nodes, differential, continuity, Hermitian spline, spaces, polynomials, approximation

In this article, an approach is implemented to include the approximation of surfaces in the theory of splines not only in E3 space but also in E^4 space. Polynomial functions are used as approximating functions in the work. The paper also developed a technique for approximating surfaces defined by a discrete set of line points using two-dimensional, three-dimensional splines on a rectangular and curvilinear grid, interpolating the values of the function and its derivatives not only at the nodes, but also on the network lines. The construction of generalized Hermitian splines in E4 space is based on the expansion of a function of three variables in the hyperplane in terms of its values and the values of its derivatives on the boundaries of the domain $\overline{\Omega}$.

In [*4], only the expansion of a function of two variables in a rectangular domain in terms of its values and the values of its derivatives on the boundaries $\overline{\Omega}$ was considered..

$\overline{\Omega} = (a_0 < x < a_1, b_0 < y < b_1, c_0 < z < c_1)$ fig.1

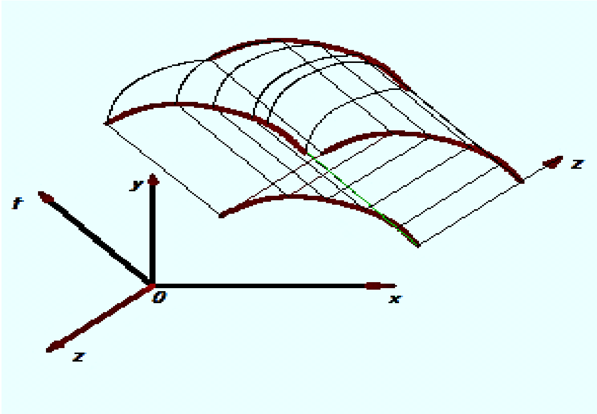

Fig.1 grid defined on $\overline{\Omega}$

Denote by $P_{k,s}(x)$ the degree polynomial $2\rho - 1$ ($\rho \in [1;\infty)$ with respect to x, through $Q_{l,t}(y)$ degree polynomial $2\rho - 1$ ($\rho \in [1;\infty)$ with respect to y and through $R_{u,v}(z)$ polynomials of the same degree with respect to Z.(s,t,v=0,1;k,l,u =0,1,…,$\rho - 1$

The coefficients of these polynomials are determined from the conditions $\frac{\partial^q P_{k,s}(x)}{\partial x^q}|_{x=a_s} = \delta_{k,q}\,\delta_{s,t}$;

$$\frac{\partial^q Q_{e,t}(y)}{\partial y^q}|_{y=b_t} = \delta_{e,q}\,\delta_{s,t};$$

$$\frac{\partial^q R_{u,v}(z)}{\partial z^q}|_{z=c_v} = \delta_{u,q}\,\delta_{s,v}; (q = 0,1,\dots,p - 1) \qquad (2.1)$$

where $\delta_{i,j}$is the Kronecker symbol defined by the equalities:

$$\delta_{i,j} = \begin{cases} 1 \text{ for } i = j \\ 0 \text{ for } i \neq j \end{cases}$$

This article discusses ways to construct generalized Hermitian splines in four-dimensional space, i.e. splines capable of interpolating a function of many variables instead of with its derivative up to a certain order, not only at the nodes, along and on the lines of the hypernet, as well as their special cases. There is a need to note that the function of many variables is considered as a geometric modeling of multifactorial processes based on a point study. Indeed, it is expedient to use such splines for mathematical description in computer-aided design systems and technological preparation of production in industries such as construction, mechanical engineering, aircraft building, shipbuilding, automotive building, turbine building, where in most practical problems of applied geometry, derivative functions that generate recoverable surfaces are not set analytically. From generalized Hermitian splines, various types of other types of splines are easily obtained, including the bicubic splines that are most often encountered in the literature. As will be shown in subsequent works of this article, splines obtained as special cases of generalized Hermitian splines. The latter circumstance makes it possible to restore surfaces with great accuracy, the skeletons of which are partially or completely specified analytically.

In this paper, the construction of generalized Hermitian splines is based on the expansion of a function of three variables in a rectangular box of the domain, in terms of its values and the values of its derivatives on the boundaries of the domain $\overline{\Omega}$

$\overline{\Omega} = (a_0 < x < a_1 \; ; b_0 < y < b_1; c < z < c_1)$ (Fig. 2)

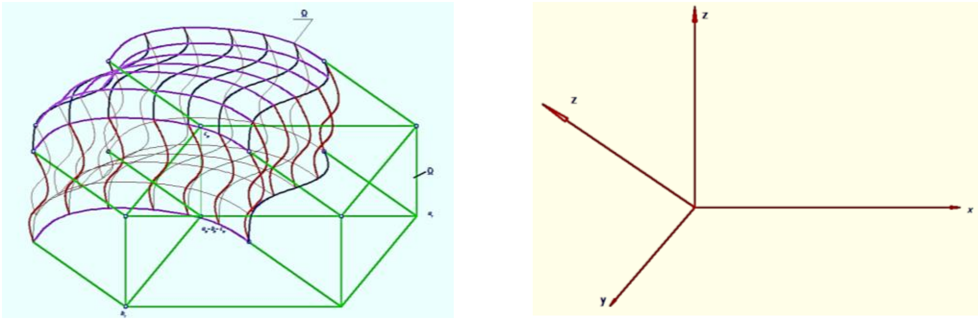

Fig. 2 spline in E^4 space

Denote by $P_{k,s}(x)$ degree polynomials $2p-1$ $(1\leq p<\infty)$ with respect to X and through $Q_{s,t}(y)$ polynomials of the same degree with respect to

$y,\ (s\ ,t=0;1\ \ k,t=\overline{0,(p-1)})\quad P_{q,e}(z)$ polynomials of the same degree with respect to Z $(q,l=0;1\ \ q,l=\overline{0,(p-1)}\ )$

Let us now assume that f(x;y;z) is an infinitely differentiable function given to the hyperplane $\overline{\Omega}$, and consider the following expressions:

The coefficients of these polynomials are determined from the conditions in E^4 by constructing axonometric projections using the generalized Hermitian spline method in E^4.

$$\left.\frac{\partial^q P_{k,s}(x)}{\partial x^q}\right|_{x=a_s}=\delta_{k,q}\,\delta_{s,t}\,\delta_{q,l};$$

$$\left.\frac{\partial^q Q_{e,t}(y)}{\partial y^q}\right|_{y=b_t}=\delta_{e,q}\,\delta_{s,t}\,\delta_{q,l}\quad q=\overline{0,(p-1)}\ ;\ l=\overline{0,(p-1)}$$

$$\left.\frac{\partial^q R_{q,l}(x)}{\partial z^q}\right|_{z=c_t}=\delta_{l,q}\,\delta_{s,t}\,\delta_{q,l}\qquad (q=0,1,....,p-1)$$

(2.2)

where $\delta_{i,q}$-

the Kronecker symbol defined by equalities

$$\delta_{i,j,k}=\begin{cases}1\ \text{for}\ i=j=k\\ 0\ \text{for}\ i\neq j=k,\ i=j\neq k\ \ \text{etc}\end{cases}$$

We now assume that f(x;y;z) is an infinitely differentiable function given by the rectangle $\overline{\Omega}$, and consider the following expressions:

$$\varphi_p(x,y)=\sum_{s=0}^{1}\sum_{k=0}^{p-1}P_{k,s}(x)f^{k,0}(a_s,y)+\sum_{t=0}^{1}\sum_{l=0}^{p-1}Q_{l,t}(y)*f^{q,l}(x,b_t)-\sum_{s,t=0}^{1}\sum_{k,l=0}^{p-1}P_{k,s}(x)\,Q_{l,t}(y)f^{k,l}(a_s,b_t);$$

$$\varphi_m(y,z)=\sum_{s=0}^{1}\sum_{k=0}^{m-1}P_{k,s}(y)f^{k,0}(a_s,y)+\sum_{n=0}^{1}\sum_{v=0}^{m-1}R_{n,v}(z)*f^{n,v}(y,l_t)-\sum_{s,t=0}^{1}\sum_{n,v=0}^{m-1}P_{k,s}(y)\,Q_{n,v}(z)f^{n,v}(b_s,s_t);$$

$$\varphi_n(x,z)=\sum_{s=0}^{1}\sum_{k=0}^{n-1}P_{k,s}(x)\,f^{k,0}(a_s,z)+\sum_{t=0}^{1}\sum_{v=0}^{n-1}R_t(z)*f^{q,v}(x,b_t)-\sum_{s,t=0}^{1}\sum_{k,l=0}^{n-1}P_{k,s}(x)\,R_{l,t}(z)f^{k,l}(a_s,c_t);\qquad(2.3)$$

Properties of polynomials in $P_{k,s}(x)$ $Q_{q,t}(y)$ $R_{u,v}(z)$,.... defined by equalities (1) ensure the fulfillment of the following boundary conditions for the function φ_p (x,y), φ_m (y,z), φ_n (x,z): orthogonal projections in the hyperplanes P_1, P_2, P_3, respectively.

$$\frac{\partial^{m+n}\varphi_p(x;y)}{\partial x^m\ \partial y^n}\Big|_{(x;y)\in\dot{\Omega}} = \frac{\partial^{m+n}f(x;y;z)}{\partial x^m\ \partial y^n}\Big|_{(x;y)\in\dot{\Omega}}$$
$$\frac{\partial^{n+k}\varphi(y;z)}{\partial y^n\ \partial z^k}\Big|_{(y;z)\in\dot{\Omega}} = \frac{\partial^{n+k}f(x;y;z)}{\partial y^n\ \partial z^k}\Big|_{(y;z)\in\dot{\Omega}}$$
$$\frac{\partial^{m+k}\varphi(x;y)}{\partial x^m\ \partial z^k}\Big|_{(x;z)\in\dot{\Omega}} = \frac{\partial^{m+k}f(x;y;z)}{\partial x^m\ \partial z^k}\Big|_{(x;z)\in\dot{\Omega}}$$
(2.4)

$(m, n, k = 0,1, \dots, p-1)$, $\dot{\Omega}$ —area border $\overline{\Omega}$.

For function φ_p(x,y), φ_p(y,z), φ_p(x,z) cthe following limit relation is true, provided that f(x;y;z) is an infinitely differentiable function.

$$\lim_{p\to\infty}\varphi_p\ (x,y) = f(x;y)\quad ((x;y)\in\overline{\Omega})$$
$$\lim_{p\to\infty}\varphi_p\ (y,z) = f(y;z)\quad ((y;z)\in\overline{\Omega})$$
$$\lim_{p\to\infty}\varphi_p\ (x,z) = f(x;z)\quad \left((x;z)\in\overline{\Omega}\right)\qquad (2.5)$$

If the function f(x;y;z) is not infinitely differentiable, then it can be approximated by the functions φ_p (x,y), φ_p (y,z), φ_p (x,z) at the final p.

In this case, we will have an estimate for the modulus of the difference:

$$|f(x;y;z) - \varphi_p(x,y)| \le \frac{S^{2P}}{2^{4P}[(2P!)]^2}\max_{(\zeta,2)\in\Omega}\left|\frac{\partial^{4P}f(\zeta,2)}{\partial\zeta^{2p}\ \partial 2^{2p}}\right|$$
$$|f(x;y;z) - \varphi_p(y,z)| \le \frac{S^{2P}}{2^{4P}[(2P!)]^2}\max_{(\zeta,2)\in\Omega}\left|\frac{\partial^{4P}f(\zeta,\delta)}{\partial\zeta^{2p}\ \partial\delta^{2p}}\right|$$
$$|f(x;y;z) - \varphi_p(x,z)|\frac{S^{2P}}{2^{4P}[(2P!)]^2}\max_{(\zeta,2)\in\Omega}\left|\frac{\partial^{4P}f(\zeta,\varphi)}{\partial\zeta^{2p}\ \partial\varphi^{2p}}\right|$$

Where S is the full lateral surface of the hyperplane $\overline{\Omega}$.

We emphasize that estimate (2.6) holds only under the condition that the function f(x; y; z) domain $\overline{\Omega}$ is 2p times continuously differentiable both in x, in y, and in z.

Let us partition the hyperplanes $\overline{\Omega}$ by the grid.

$\Delta_{n,m,k}$: $a_0 = x_0 < x_1 < \dots < x_{n-1} < x_n = a_1$

$b_0 = y_0 < y_1 < \dots < y_{m-1} < y_m = b_1$

$c_0 = z_0 < z_1 < \dots < z_{k-1} < x_k = c_1$

(2.7)

In doing so, we will denote $h_j = y_{j+1} - y_j$, $h_k = z_{k+1} - z_k$

The mesh (2.7) splits the hyperplanes $\overline{\Omega}$ into hyperplanes

$\Omega_{i,j,k} = [\,x_i \le x \le x_{i+1}, y_i \le y \le y_{j+1}, z_k \le z \le z_{k+1}]$

In each of the hyperplanes, we approximate the function f(x;y;z) by the functions $\varphi_p(x,y), \varphi_p(x,z), \varphi_p(y,z)$, for example when P = 2. In this case, the index P will be omitted, and in its place we will put the index i,j,k denoting that the function

$\varphi_{i,j,k}(x,y,z)$ refers to a grid cell $\overline{\Omega}_{i,j,k}$, being its carrier.

In this case, $\varphi_{i,j,k}(x,y,z)$ takes the form for two-dimensional projection spaces, we note $\varphi_{i,j}(x,y), \varphi_{j,k}(y,z), \varphi_{i,k}(x,z)$:

$$\varphi_{i,j}(x,y) = \sum_{k,s=0}^{1} P_{k,s}(x) f^{k,0,}(x_{i+s,}\, y) + \sum_{l,t=0}^{1} Q_{l,t}(y) f^{0,l}(x_{,}\, y_{j+t}) - $$
$$-\sum_{s,k,l,t=0}^{1} P_{k,s}(x) Q_{l,t}(y) f^{k,l}(x_{i+s,}\, y_{j+t});$$

$$\varphi_{j,k}(x,z) = \sum_{k,s=0}^{1} P_{k,s}(x) f^{k\,0,}(x_{k+i,}\, y) + \sum_{u,v=0}^{1} Q_{l,t}(y)\ f^{0,u}(x_{,}\, z_{k+v}) - \sum_{k,s,u,v=0}^{1} P_{k,s}(x) Q_{u,v}(y)\ f^{s,v}(x_{i+s}\, z_{j+u});$$
(2.6)

$$\varphi_{i,k}(y,z) \sum_{k,s=0}^{1} P_{k,s}(y) f^{k,0}(y_{k+i,}\, z) + \sum_{u,v=0}^{1} Q_{u,v}(z)\ f^{0,v}(y_{,}\, z_{j+v}) - \sum_{k,s,u,v=0}^{1} P_s(y) Q_{u,v}(z)\ f^{k,u}(y_{i+k}\, z_{j+t}).\qquad (2.8)$$

For P =3 you can give the following form:

$P_{0,0}(x) = -\frac{(x-x_{i+1})^2}{h_i^3}$ (3x; - 2x - x_{i+1});

$P_{0,1}(x) = -\frac{(x-x_i)^2}{h_i^3}$ ($3x_{i+1}$; - 2x - x_i);

$P_{1,0,0}(x) = -\frac{(x-x_i)^2}{h_i^2}$ (x_i - x_{i+1}); where $h_i = x_{i+1} + x$;

$P_{1,1,0}(x) = -\frac{(x-x_i)^2}{h^2}$ (x- x_{i+1}); (*2.9)

$P_{0,0,0}\ (x) = -\frac{(x-x_i)^2}{h_i^3}\ (3x_{i+1}; -2x - x_i);$

$P_{0,0,1}\ (x) = -\frac{(x-x_{i+1})^2}{h_i^3}\ (3x_i - 2x - x_{i+1});$

$P_{0,1,1}\ (x) = -\frac{x-x_i}{h_i^2}\ (x - x_{i+1})^2;$

$P_{1,1,1}\ (x) = -\frac{(x-x_i)^2}{h_i^2}\ (x_i - x_{i+1});$ where $h_i = x_{i+1} - x_i$; (2.11)

$P_{0,0,0}\ (y) = -\frac{(y-y_{j+i})^2}{h_i^3}\ (3y_j - 2y - y_{j+i});$

$P_{0,1,0}\ (y) = -\frac{(y-y_{j+i})^2}{h_i^3}\ (3y_j - 2y - y_{j+i});$

$P_{1,0,0}\ (y) = -\frac{y-y_j}{h_i^2}\ (y - y_{j+i})^2;$

$P_{1,1,0}\ (y) = -\frac{(y-y_j)^2}{h_i^2}\ (y - y_{j+1});$ where $h_j = y_{i+1} - y_j$ (2.9)

$Q_{0,0,0}\ (y) = -\frac{(y-y_{j+i})^2}{h_i^3}\ (3y_j - 2y - y_{j+i});$

$Q_{1,0,0}\ (y) = -\frac{y-y_j}{h_i^2}\ (y - y_{j+i})^2;$

$Q_{0,0,1}\ (y) = -\frac{(y-y_{j+i})^2}{h_i^3}\ (3y_{j+1} - 2y - y_{j+1});$

$Q_{0,1,1}\ (y) = -\frac{(y-y_{j+i})^2}{h_i^3}\ (3y_{j+1} - 2y - y_{j+1});$

$Q_{0,1,1}\ (y) = -\frac{(y-y_{j+i})^2}{h_i^3}\ (3y_{j+1} - 2y - y_{j+1});$

$Q_{1,1,1}\ (y) = -\frac{(y-y_j)^2}{h_k^2}\ (y - y_{j+1});$ where $h_j = y_{i+1} - y_j$ (2.10)

$R_{0,0,0}\ (z) = -\frac{(z-z_{k+i})^2}{h_k^3}\ (3z_k - 2z - z_k);$

$R_{0,0,1}\ (z) = -\frac{(z-z_{k+i})^2}{h_k^3}\ (3z_k - 2z - z_k);$

$R_{0,1,1}\ (z) = -\frac{(z-z_{k+i})^2}{h_k^3}\ (3y_{k+1} - 2z - z_k);$

$R_{1,0,1}\ (z) = -\frac{z-z_k}{h_i^2}\ (z - z_{k+1})^2;$

$R_{1,1,1}\ (z) = -\frac{(z-z_k)^2}{h_k}\ (z - z_{k+1});$ where $h_k = z_{k+1} - z_k$ (2.11)

Similarly, one can write a three-dimensional spline, which is an analogue of the bicubic Hermitian spline.

It will look like:

$$\varphi\ (x,y,z) = \sum_{i=0}^{n-1}\sum_{j=0}^{m-1}\sum_{k=0}^{l-1} \varphi_{i,j,k}\ (x,y,z) \qquad (2.12)$$

Spline(2.12), like the bicubic Hermitian spline, belongs to the function space $C^{1,1,1}(\overline{\Omega})$, but in contrast to it, the interpolation of the function f (x, y, z) is performed not only at the nodes, but also on the grid lines. [6,8,9]

Indeed, from the properties $P_{i,j}(x)$, $Q_{j,k}(y)$, $Q_{i,k}(z)$ determined by the conditions, we get the equalities $(x_i, y, z) = f\,(x_i, y, z)$, $i = \overline{0,n}$, $\varphi\,(x, y_j, , z) = f\,(x, y_j, z)$, $j = \overline{0,m}$,

$\varphi\ (x, y, z_k) = f\,(x, y, z_k)$, $k = \overline{0,q}$, ,

$\frac{\partial\varphi\ (x,y,z)}{\partial x}|_{x=x_i} = \frac{\partial f(x,y,z)}{\partial x}\ |_{x=x_i};$

$\frac{\partial\varphi\ (x,y,z)}{\partial x}|_{y=y_j} = \frac{\partial f(x,y,z)}{\partial y}|_{y=y_j};$

$\frac{\partial\varphi\ (x,y,z)}{\partial z}|_{z=z_k} = \frac{\partial f(x,y,z)}{\partial z}\ |_{z=z_k};$

(2.13)

In addition, at the nodes of the grid of the hyperplane, the equality also holds

mixed derivatives:

$$\frac{\partial^3\varphi\ (x,y,z)}{\partial x\,\partial y\,\partial z}\Big|_{\substack{x=x_i\\ y=y_j\\ z=z_k}} = \frac{\partial^3 f(x,y,z)}{\partial x\,\partial y\,\partial z}\ \Big|_{\substack{x=x_i\\ y=y_j\\ z=z_k}}$$

Estimate (2.5) for the reconciler to the modulus of the difference $|f(x,y,z) - \varphi\ (x,y,z)|$ takes the form:

$$|f\ (x,y,z) - \varphi\ (x,y,z)| \le \max_{i,j,k}\left\{\frac{S^{4P}}{2^{4P}(2_P\,!)^2}\ \max_{(\zeta,\eta,\psi)\,\epsilon\,\Omega}\ \left|\frac{\partial^{4P} f\ (\zeta,\eta,\psi)}{\partial_\zeta^{2p}\partial_\eta^{2p}\partial_\psi^{2p}}\right|\right\} \quad (2.14)$$

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